

Fatigue damage assessment for Sine-on-Random: a comparative analysis of the predictive capability and application limits of spectral methods

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ABSTRACT. Mechanical and electronic components, mainly in aerospace and rotating machinery environments, are generally subjected to Sine-on-Random (SoR) excitations, where deterministic sinusoidal tones are superimposed on a broadband random vibration. While international testing standards require the validation of components under these complex profiles, the numerical evaluation of fatigue damage poses a major analytical challenge. The time-domain rainflow counting method is computationally prohibitive. Conversely, frequency-domain spectral methods rely on Gaussian assumptions that are explicitly violated by SoR loads, leading to inaccurate damage estimates. To bridge this gap, researchers have developed two main strategies: practical approaches that synthesize an equivalent random Power Spectral Density (PSD) based on damage or energy equivalence, and rigorous theoretical methods deriving analytical cycle distributions. This paper evaluates and compares the available methodologies, aiming to highlight the benefits and limitations of each. Ultimately, this comparative study serves as a practical guide for engineers to select the most appropriate tool for fatigue damage assessment under SoR vibrations.

KEYWORDS. Vibration fatigue, Sine on Random, Structural dynamics, Non-Gaussian vibrations



INTRODUCTION

In practical engineering scenarios, mechanical and electronic components are rarely subjected to purely stochastic or purely sinusoidal excitations. Most real-life environments are characterized by "mixed-mode" conditions, whereby deterministic and random components coexist. The most notable example of this phenomenon is the Sine-on-Random (SoR) profile, in which one or more sinusoidal tones, representing harmonic components, are superimposed on a broadband Gaussian random vibration background.

The aerospace sector is the primary field of application for SoR profiles. In general aircraft environments, excitation is characterized by a complex combination of sources. The broadband random component is primarily generated by aerodynamic turbulence, boundary layer fluctuations, and jet engine exhaust noise. Concurrently, harmonic components are produced by various rotating masses. In the case of rotary-wing aircraft, these are generated by the main and tail rotors and transmission shafts. In fixed-wing propeller aircraft, they stem from propeller rotation, whereas in jet aircraft, they arise from high-speed turbine stages and auxiliary power units.

The development of SoR test specifications, as outlined in international standards such as MIL-STD-810 [1] and RTCA DO-160 [2], is rooted in the historical necessity to validate components subject to mixed loads. The emphasis of these standards is on replicating the actual effects of environmental conditions on equipment, as opposed to the mere imitation of the signals themselves [3]. While physical testing on electrodynamic shakers remains one of the primary methods for verifying that a component can survive these complex environments, these regulatory profiles are also fundamental during the design phase. Designers employ these standardized loads to perform design verification, ensuring that the structural integrity of a component is sufficient to withstand the prescribed mission profiles, with a specific focus on mechanical fatigue resistance [4].

However, evaluating fatigue damage under SoR conditions poses substantial analytical challenges. Given the inherently non-Gaussian nature of SoR loads, the prevailing approach for assessing fatigue damage is the time-domain method, which is based on the rainflow counting algorithm [5]. This approach is considered the "gold standard" as it directly counts stress cycles from the signal's time history without requiring statistical assumptions. However, it is widely acknowledged that time-domain analysis is extremely computationally expensive, which makes its implementation prohibitive for long-duration missions or complex structural models.

For this reason, in environments characterized by purely random vibrations, frequency-domain spectral methods (such as those developed by Dirlik [6] or Rayleigh [7]) are preferred due to their computational efficiency. These classical methods, however, rely on the strong assumption of a stationary Gaussian process [8]—an assumption that is explicitly violated in Sine-on-Random scenarios [9]. The presence of deterministic tones has been shown to fundamentally alter the probability density function (PDF) of the stress ranges, resulting in inaccurate damage estimates if standard Gaussian formulas are applied [9-10].

To leverage the convenience offered by frequency-domain methods for SoR loads, researchers have proposed two primary strategies: a rigorous, theoretical approach and a more practically oriented engineering approach. The theoretical approach focuses on deriving an analytical PDF of the cycles that accounts for the combined sine-plus-noise statistics. The foundational work of Rice [10] established the theoretical basis for this line of thought, though its practical applications are often constrained to specific, stringent cases, such as when the sine frequency is in proximity to the central frequency of the random noise. However, in numerous structural dynamics applications, the sine waves are positioned well beyond the random band, thereby constituting a perpetual violation of Rice's hypotheses [1,2].

Consequently, practical engineering approaches have become increasingly widespread in industrial applications. These methodologies rely on generating an equivalent random Power Spectral Density (PSD) that represents the SoR phenomenon through a damage-based or energy-based equivalence. Authors such as Lalanne [11], Irvine [12-14], and Jang [15] have developed innovative methods to synthesize a random profile that produces equivalent fatigue damage. In contrast, Cho [16] proposed a method that synthesizes a random profile with the same Root Mean Square (RMS) level as the original SoR signal. These approaches are straightforward to apply; once the equi-damaging or equi-RMS PSD is obtained, the SoR process can be treated as a pure Gaussian random vibration. Thus, the equivalent PSD can be analysed using standard spectral methods [17], effectively transforming the complex mixed-mode problem into a conventional Gaussian scenario.

Given the complexity of the Sine on Random phenomenon, the objective of this work is to provide the scientific and engineering communities with a comprehensive knowledge of the problem and the approaches available in the literature. Specifically, the main contributions of the paper consist of the following:



- This work reviews the theoretical notions regarding random vibration fatigue and statistics of Sine on Random signals, which are considered necessary for the understanding of the presented Sine on Random fatigue assessment approaches.
- The paper concretely gathers in a single manuscript all the available fatigue assessment methodologies for SoR environments, providing a unified review and classifying them according to their characteristics.
- The presented approaches are subjected to a systematic numerical comparison based on identical benchmark loading configurations, which go from simple single-sinusoid profiles with rectangular PSDs, where the sinusoidal frequency is shifted relative to the random background, to a real MIL-STD SoR profile, consisting of a broadband random PSD and four sinusoidal signals with different frequencies and different amplitudes.
- The paper concludes with a discussion concerning the practical applicability and main limitations of each approach, providing engineers with pragmatic guidelines for selecting the most fitting method based on the loading scenario.

RANDOM FATIGUE

This section outlines the established analytical approaches for the fatigue life assessment of components subjected to purely random Gaussian loadings.

Spectral methods for fatigue life estimation

Under the assumption of Gaussian distribution, any random process $x(t)$ can be described in the frequency domain by its statistical properties, namely its PSD $S_{xx}(\omega)$, which is defined as the Fourier transform of the autocorrelation function $R_{xx}(\tau) = E[x(t)x(t+\tau)]$ [18]:

$$S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-i\omega\tau} d\tau \quad (1)$$

Two definitions of PSD exist, the double-sided $S_{xx}(f)$ and the single-sided $G_{xx}(f)$, and the relation is provided in Eqn. (2):

$$G_{xx}(f) = 2S_{xx}(f) \quad (2)$$

The spectral moments m_n are important characteristics of a random process; they are defined as:

$$m_n = \int_0^{\infty} \omega^n G_{xx}(\omega) d\omega \quad (3)$$

where the subscript n represents the order. Spectral moments represent some important properties of the signal; for example, the moment of order zero, denoted as m_0 , coincides with the variance of the random signal [19].

Combinations of spectral moments of higher order are used to characterize some statistical properties of the random signal that are of interest in fatigue analysis. For a Gaussian process, the number of zero-upcrossings per second (v_0^+) and the peak rate (v_p) are defined in Eqns. (4) and (5):

$$v_0^+ = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \quad (4)$$

$$v_p = \frac{1}{2\pi} \sqrt{\frac{m_4}{m_2}} \quad (5)$$



Other crucial spectral moments combinations are the bandwidth parameter ϵ , which quantifies the spectral spread of the process, and the irregularity factor γ , which serves as an alternative measure of the process bandwidth. They are defined as follows:

$$\epsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \quad (6)$$

$$\gamma = \frac{m_2}{\sqrt{m_0 m_4}} \quad (7)$$

The bandwidth parameter ranges from 0 for a strictly narrow-band (NB) process to 1 for a wide-band (WB) process, whereas, by definition, γ approaches unity for NB conditions and tends to zero for WB signals.

The distribution of maxima (or peak values x_p) $p_{x_p}(x, \epsilon)$ for a zero mean Gaussian process depends on this parameter and follows the formulation provided by Cartwright and Longuet-Higgins [20] and represented, in a normalized form, in Eqn. (8):

$$p_\zeta(\zeta) = \frac{1}{(2\pi)^{1/2}} \left[\epsilon^{-\frac{\zeta^2}{2\epsilon^2}} + \left(1 - \epsilon^2\right)^{\frac{1}{2}} \zeta e^{-\frac{\zeta^2}{2}} \int_{-\infty}^{\frac{\zeta(1-\epsilon^2)^{\frac{1}{2}}}{\epsilon}} e^{-\frac{x^2}{2}} dx \right] \quad (8)$$

where $\zeta = x_p / \sqrt{m_0}$ represents the *normalized maxima* and $\sqrt{m_0}$ is the RMS value of the Gaussian process.

The fatigue characterization of materials is fundamentally described by the S-N curve, often expressed through the power-law relationship also known as Basquin's law [21]:

$$N_s^m = C \quad (9)$$

where N denotes the number of cycles to failure at a constant stress amplitude s , while C and m represent material-specific fatigue parameters.

For components subjected to stochastic excitations, fatigue damage is typically evaluated under the assumption of linear damage accumulation, as defined by the Palmgren-Miner rule [22]. Under this framework, the expected damage $E[D]$ accumulated per unit time, relative to the material's fatigue properties, is expressed by the following generalized integral [23]:

$$E[D] = \frac{\nu_p}{C} \int_0^\infty s^m p_s(s) ds \quad (10)$$

where ν_p represents the expected number of cycles per unit time and $p_s(s)$ is the PDF of the stress cycle amplitudes, with s denoting the stress cycle amplitude.

In cases where the S-N curve intercept constant C is unknown, it may be normalized to unity. This modifies Eqn. (10) into a metric defined as *pseudo-damage*, which remains a valid basis for the comparative assessment of distinct loading histories.

While the PDF of stress cycles can be precisely extracted from time-history data using the rainflow counting (RFC) algorithm, such procedures are computationally intensive, especially for long-duration or high-frequency signals. To overcome these limitations, various spectral methods have been developed to estimate the cycle amplitude PDF directly from the PSD of the load. In this context, the damage is treated as an expected value, as the stochastic nature of the process is captured through the spectral moments of the PSD.

The complexity of the spectral characterization depends on the bandwidth of the process. NB processes can be effectively approximated as quasi-sinusoidal signals, where the distribution of peaks is statistically described by the zero-th spectral



moment m_0 and follows a Rayleigh distribution [7]. Conversely, WB and bimodal excitations, typical of complex structural responses, require higher-order spectral moments to accurately characterize the irregularity of the signal and the presence of multiple frequency components [17]. Among the most robust and widely adopted methods for broad-band processes are Dirlik's empirical closed-form solution [6] or the Tovo-Benasciutti method [24].

Fatigue damage spectrum

The Fatigue Damage Spectrum (FDS) is a useful tool in engineering practice since it provides a frequency-domain representation of the expected fatigue damage, thereby enabling a direct comparison between different types of excitations, whether deterministic, impulsive, or stochastic, by reducing them to their cumulative effect on a simplified mechanical model (single degree of freedom system)[25-26].

Since the FDS has been used by some authors as a tool to obtain an equivalent random PSD from a SoR specification, it is fundamental to describe the FDS calculation both in the time domain and in the frequency domain.

Definition and Fundamentals. The FDS is an essential tool for evaluating the damaging potential of a vibration signal. It is defined as the envelope of the fatigue damage experienced by a set of Linear Time-Invariant (LTI) Single-Degree-of-Freedom (SDOF) systems. These systems are characterized by a varying natural frequency f_0 (or angular frequency $\omega_0 = 2\pi f_0$) and a constant damping ratio ξ (often expressed via the quality factor $Q = \frac{1}{2\xi}$). The spectrum is calculated for a specific fatigue exponent m , corresponding to the slope of the material's S-N curve.

Time-Domain Calculation Procedure. Regardless of the signal's nature, the FDS can be computed directly from time-history data. The computational procedure for generating the spectrum follows these fundamental steps:

- *System Response:* The response of each SDOF system is calculated in terms of relative displacement $Z(t)$.
- *Cycle Extraction:* The stress cycle histogram is extracted from the response using the RFC algorithm. For SDOF systems with high Q factors, the response is inherently NB, and the distribution of cycles tends to align with the distribution of the peaks Z_p .
- *Damage Accumulation:* Linear damage accumulation is performed according to the Palmgren-Miner rule:

$$D = \frac{K^m}{C} \sum_{i=1}^{N_{bin}} n_i Z_{p,i}^m \quad (11)$$

where n_i is the number of cycles at amplitude $Z_{p,i}$, C is the fatigue strength coefficient, and K is the proportionality factor between displacement and stress ($s_{inst}(t) = K \cdot Z(t)$, with s_{inst} denoting stress instantaneous values).

By assuming a unit proportionality factor ($K = 1$) and normalizing the fatigue strength coefficient ($C = 1$), Eqn. (11) yields a displacement-based pseudo-damage metric. This approach allows for a relative comparison of the fatigue damage potential between different load histories when C and K are unknown.

By repeating this calculation for a defined range of natural frequencies, f_0 , the function $D(f_0)$, known as the Fatigue Damage Spectrum, is obtained.

FDS of a random signal in the frequency domain. For Gaussian and stationary random processes, the analytical expression for the PDF of the response peaks for an SDOF system is well-defined. This allows for the determination of the FDS directly from the spectral content, significantly reducing the computational effort compared to time-domain simulations. Since such signals are entirely described by their PSD, the FDS can be evaluated through the following analytical procedure:

- *Spectral Density:* Calculation of the vibratory signal's PSD or reference to PSDs taken from regulations.
- *Spectral Moments:* Determination of spectral moments and RMS values σ_Z , $\sigma_{\dot{Z}}$, and $\sigma_{\ddot{Z}}$ of the relative displacement of the SDOF system response, along with its velocity and acceleration. For zero-mean signals, these moments correspond to the RMS values.

- **Spectral Parameters:** Given the Gaussian nature of the input, it is assumed that the response is also Gaussian. Consequently, the zero up-crossing rate (Eqn. (4)) and the peak rate (Eqn. (5)) of the response are calculated. Additionally, the irregularity factor, as defined in Eqn. (7), is calculated.
- **Peak Distribution:** Definition of the PDF of the response peaks.

Although the distribution of rainflow counting cycles is essential for an accurate fatigue damage estimation, the response peaks PDF remains a fundamental tool for NB signals [27]. The response of an SDOF system to a Gaussian excitation is characterized by a Gaussian signal concentrated around the system's natural frequency. As the damping coefficient decreases, the response becomes increasingly NB. Under these conditions, the response $Z(t)$ can be considered a NB signal, allowing the peak distribution to be approximated by a Rayleigh distribution. Under such conditions, the FDS for a Gaussian random signal can be evaluated using the following expression:

$$D(f_0) = K^m \frac{v_p T}{C} \sigma_z^m 2^{\frac{m}{2}} \Gamma\left(\frac{m}{2} + 1\right) \quad (12)$$

where σ_z is the root-mean-square value of the response, T is the exposure time, and Γ is the gamma function.

Evaluating the fatigue spectrum in the frequency domain provides the expected value of the damage, effectively eliminating the intrinsic statistical variability that arises from calculations based on individual time histories. This method ensures high computational efficiency while maintaining excellent agreement with time-domain results.

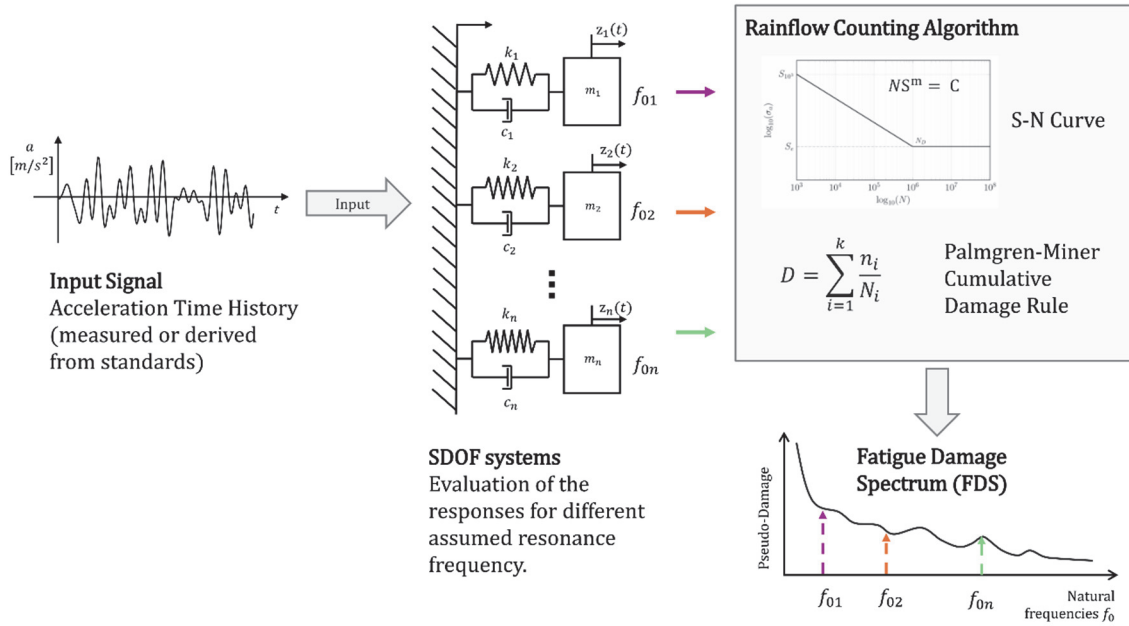


Figure 1: Overview of the FDS evaluation process in time domain.

The White Noise Approximation. A widely adopted simplification is the White Noise Approximation. By assuming that the input PSD, $G_{xx}(f)$, is locally constant around the natural frequency f_0 , the RMS stress response can be directly estimated as [19]:

$$\sigma_z = \sqrt{\frac{G_{xx}(f_0) Q}{4(2\pi f_0)^3}} \quad (13)$$

Substituting this into Eqn. (12) yields the analytical closed form for the FDS for the white noise case:



$$FDS(f_0) = K^m \frac{f_0 T}{C} \left[\frac{QG_{xx}(f_0)}{2(2\pi f_0)^3} \right]^{\frac{m}{2}} \Gamma\left(\frac{m}{2} + 1\right) \quad (14)$$

In comparative studies, it is also possible to set $K=1$ and $C=1$ for comparative purposes between different loadings. A schematic representation of the FDS evaluation process in the time domain is provided in Fig. 1.

STATISTICAL CHARACTERISTICS OF SINE-ON-RANDOM SIGNALS

In this section, considerations are made about the statistics of mixed SoR signals and the inapplicability of conventional spectral methods in this context.

A Sine-on-Random signal represents a mixed-mode excitation where one or more deterministic sinusoidal components are superimposed on a stochastic Gaussian process. In its simplest form, a single-tone SoR signal $z(t)$ can be expressed as the sum of a deterministic sine wave $y(t)$ and a random Gaussian process $x(t)$:

$$z(t) = x(t) + y(t) \quad (15)$$

Statistics of the Sine Component

This section draws upon the seminal work of S.O. Rice [10], who provided the analytical foundations for the study of random processes and signal theory.

A sinusoid is defined by the time-dependent function $y(t) = a \sin(\omega_s t + \psi)$, where a is the amplitude, ω_s is the angular frequency, and ψ is the phase. By assuming the argument $\theta = \omega_s t + \psi$ to be a random variable uniformly distributed over $[0, 2\pi]$, the sine wave $y = a \sin(\theta)$ represents a transformation of a random variable and, consequently, is a random variable itself.

Since every random variable is uniquely characterized by its PDF and an associated characteristic function ϕ^1 , the sinusoid can be described analytically through ϕ_y [1]:

$$\phi_y(\eta, a) = E[e^{i\eta y}] = \frac{1}{2\pi} \int_0^{2\pi} e^{i\eta a \sin(\theta)} d\theta = J_0(a\eta) \quad (16)$$

where J_0 is the Bessel function of order zero and η is the transform variable associated with the characteristic function of the random variable y . Starting from ϕ_y , the PDF of the instantaneous values is derived via inverse Fourier transform:

$$p_y(y, a) = \begin{cases} \frac{1}{\pi} (a^2 - y^2)^{-1/2}, & |y| \leq a \\ 0, & |y| > a \end{cases} \quad (17)$$

As shown in Fig. 2, the sine wave spends most of its time near the extremes $\pm a$. Even though the PDF of instantaneous values is necessary to describe the process, the statistics of extreme values are very useful in some applications such as fatigue and structural dynamics. The PDF of its maxima (peaks) p_{y_p} is represented by a Dirac Delta function centred at the amplitude value a [28,29]:

¹ The characteristic function $\phi_x(\eta)$ of a PDF $p_x(x)$ is defined as its Fourier transform:

$$\phi_x(\eta) = \mathcal{FT}[f_x(x)] = \int_{-\infty}^{\infty} p_x(x) e^{i\eta x} dx$$

The functions $p_x(x)$ and $\phi_x(\eta)$ thus constitute a *transform pair*.



$$p_{y_p}(y, a) = \delta(y - a) \quad (18)$$

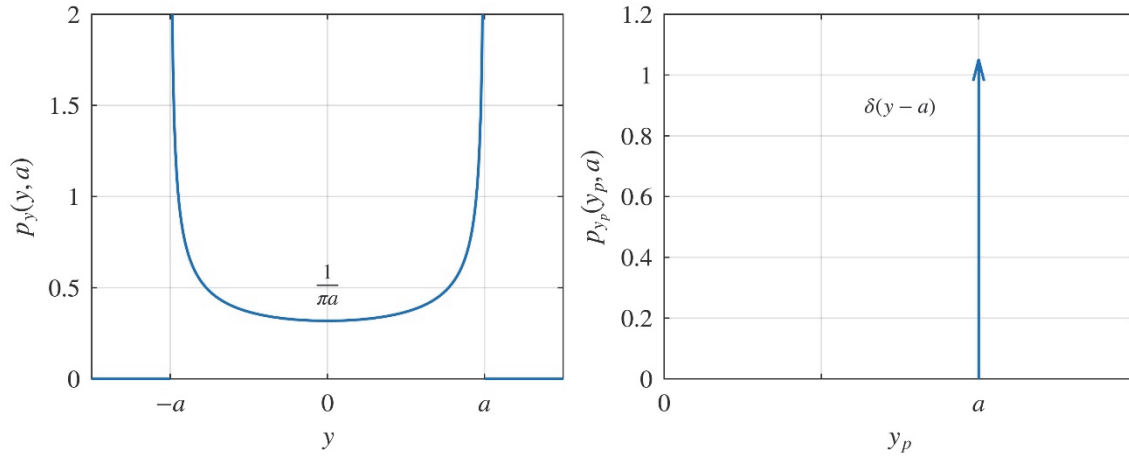


Figure 2: Distributions of a Sine Wave: Instantaneous values (left) and peak values (right).

Statistics of the Random Component

Assuming a stationary Gaussian process $x(t)$, the PDF of the background noise, $p_x(x, \mu_x, \sigma_x)$ is represented by a Gaussian function, and its characteristic function is:

$$\phi_x(\eta, \mu_x, \sigma_x) = \exp\left(i\mu_x\eta - \frac{\sigma_x^2\eta^2}{2}\right) \quad (19)$$

where μ_x and σ_x are, respectively, the mean value and the standard deviation of the random signal. It must be noted that, throughout this work, random signals will be assumed having $\mu_x = 0$. This means that σ_x coincides with the signal's RMS.

Combined SoR statistics

A SoR signal can be analytically treated as the superposition of two independent random variables. The PDF of the instantaneous values of the combined signal, $z(t) = x(t) + y(t)$, is defined by the convolution of the Gaussian PDF and the sine PDF [10]:

$$p_z(z) = p_x(x) * p_y(y) = \int_{-\infty}^{+\infty} p_x(z-b) p_y(b) db \quad (20)$$

To simplify the evaluation of this convolution integral, the characteristic functions ϕ_x and ϕ_y are involved. According to the properties of Fourier Transform (FT) pairs, the FT of the PDF of a sum of independent variables corresponds to the product of their respective characteristic functions [10]. Given the Gaussian nature of the random noise with zero mean ($\mu_x = 0$) and standard deviation σ_x (representing the RMS value of the stochastic component), the characteristic functions for the two components are expressed in Eqns. (16) and (19). The resulting characteristic function of the SoR signal is therefore $\phi_z(\eta) = \phi_x(\eta) \cdot \phi_y(\eta)$. The PDF of the process is then recovered through the inverse Fourier transform of $\phi_z(\eta)$, thereby avoiding the direct evaluation of the convolution integral in (20).

The statistical behaviour of the combined process is heavily influenced by the ratio a/σ_x , which represents the amplitude of the sinusoidal tone relative to the RMS level of the background noise. As this ratio increases, the distribution shows a progressive deviation from the Gaussian bell shape, as illustrated in Fig. 3. The SoR PDF gradually adopts a bimodal profile, which is characteristic of the sinusoidal component, resulting in a significantly higher probability density at the tails of the distribution. This non-Gaussian nature is of critical importance in estimating fatigue life, as it directly affects the distribution



of peak values and the subsequent cumulative damage evaluation [2]. Due to the non-Gaussianity of the process, the use of standard spectral methods, such as Dirlik's or narrow-band approximation, will lead to inaccurate life estimations [8].

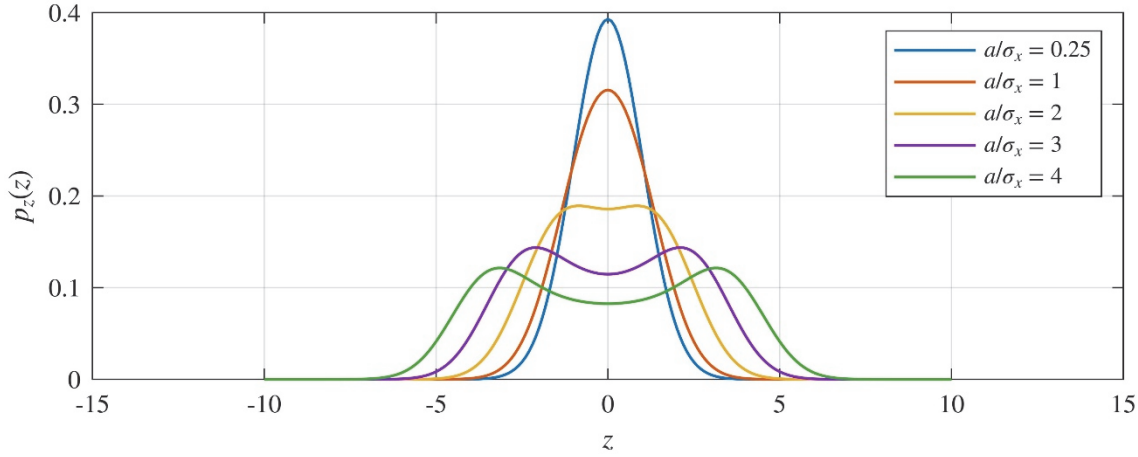


Figure 3: Distribution of instantaneous values of a single sinusoid SoR signal with varying a / σ_x ratio.

AVAILABLE APPROACHES FOR SoR FATIGUE DAMAGE EVALUATION

To overcome the limitations of classical spectral methods imposed by the non-Gaussian nature of SoR signals, the literature proposes two primary frameworks: the theoretical derivation of the cycle PDF and the practical derivation of an equivalent PSD.

Cycle PDF equivalence: Rice's method

Originally pioneered by S.O. Rice [10], this theoretical approach focuses on the rigorous derivation of the PDF of the stress cycle amplitudes. It treats Sine-on-Random excitation as a distinct stochastic process, explicitly modelling the interaction between the deterministic harmonic components and the background Gaussian noise.

The fundamental objective of this approach is to estimate the PDF of the rainflow cycle amplitudes directly in the frequency domain based exclusively on statistical properties. Rice's approach avoids the forced Gaussian assumption, thereby preserving the inherent non-Gaussian characteristics of the SoR signal. These characteristics include the unique peak distribution and the increased probability of high-amplitude cycles.

For a NB random process overlaid with N_b harmonics, Rice's PDF for *peak values* s_p is expressed as follows:

$$p_{s_p}(s_p) = s_p \int_0^\infty x J_0(s_p \cdot x) e^{-\frac{\sigma_{s,r}^2 \cdot x^2}{2}} \prod_{i=1}^{N_b} J_0(a_{s_i} \cdot x) dx \quad (21)$$

where $\sigma_{s,r}^2$ is the variance of the zero-mean random component, a_{s_i} ($i=1,2,\dots,N_b$) are the sine amplitudes and J_0 is the Bessel function of order zero.

The only case in which Eqn. (21) offers a closed-form solution is the single-tone scenario: a single sinusoidal signal ($N_b = 1$) of amplitude a_s superimposed on a random PSD. The resulting PDF takes the following shape:

$$p_{s_p}(s_p) = \frac{s_p}{\sigma_{s,r}^2} e^{-\frac{s_p^2 + a_s^2}{2\sigma_{s,r}^2}} I_0\left(\frac{a_s s_p}{\sigma_{s,r}^2}\right) \quad (22)$$

where I_0 represents the modified Bessel function of order zero.

Once the PDF is known, the evaluation of fatigue damage can proceed by means of Miner's integral linear damage evaluation formula (Eqn. (10)). If the random component of the SoR process is NB, the SoR peak values PDF $p_{s_p}(s_p)$ represents a



good approximation of the distribution of rainflow counted cycles $p_s(s)$. It should be noted, however, that the accuracy of this approximation diminishes as the bandwidth of the random component increases.

While a closed-form solution exists for a single sinusoid, the integration of Eqn. (21) must be performed numerically when dealing with multiple pure tones ($N_b > 2$), possibly leading to a considerable increase in computational times.

A further limitation in Rice's theoretical framework is the requirement for incommensurate harmonic frequencies. For the sine phases under consideration to be treated as independent and uniformly distributed, it is necessary that the frequencies are not rational multiples of one another. However, in practical engineering scenarios, particularly those involving rotating machinery, pure tones are frequently harmonics (integer multiples) of the fundamental rotation speed. This discrepancy between theoretical assumptions and real-world specifications may limit the statistical representativeness of the analytical model in certain operational environments.

For this set of reasons, practical rather than purely theoretical methods will be introduced.

Although these approaches may sacrifice some degree of accuracy, they offer greater ease of implementation and significantly reduced computational time for fatigue life estimation.

Equivalent PSD approaches

The primary objective of these practical approaches is to simplify the complex SoR environment by defining a purely stochastic PSD that yields the same damage potential as the original mixed-mode signal. Despite its inherent approximation, as it imposes a non-Gaussian SoR distribution onto a Gaussian framework, this methodology is employed due to its notable computational efficiency. The inevitable loss of physical and statistical fidelity is balanced by the significant reduction in processing time compared to exhaustive time-domain analyses, facilitating the rapid synthesis of qualification test profiles. The introduction of the FDS of SoR loadings is necessary to understand the basis of some of the following approaches.

Fatigue damage spectrum of a Sine-on-Random signal. While damage evaluation for purely Gaussian or purely sinusoidal environments is well-established, the mixed nature of SoR processes requires a dedicated methodological approach, as described in the works of Angeli et al. [30] and Lalanne [11].

FDS for signals with a single sinusoid. Consider a SoR acceleration profile defined by a single sinusoidal tone (with amplitude a and frequency f_s) superimposed on a Gaussian stochastic process described by its PSD $G_{xx}(f)$. Due to the harmonic component, the signal deviates from a Gaussian distribution, leading to the need for a method for estimating the cycle distribution.

The first step involves determining the response of each SDOF system in terms of relative displacement $Z(t)$. The mean square value of the random component of the response, $\sigma_{Z,r}^2$, can be calculated as:

$$\sigma_{Z,r}^2 = \int_{-\infty}^{\infty} \left| H_{Z/x}(f) \right|^2 G_{xx}(f) df \quad (23)$$

Alternatively, using the white noise approximation:

$$\sigma_{Z,r} = \sqrt{\frac{Q \cdot G_{xx}(f_0)}{4 \cdot (2\pi f_0)^3}} \quad (24)$$

The amplitude of the sinusoidal response Z_s is determined via the Frequency Response Function (FRF):

$$Z_s = \left| H_{Z/x}(f_s) \right| a = \frac{a}{(2\pi f_0)^2 \sqrt{\left[1 - \left(\frac{f_s}{f_0} \right)^2 \right]^2 + \left(\frac{f_s}{Q f_0} \right)^2}} \quad (25)$$



The RMS values for the sinusoidal response and its derivatives ($\dot{Z}_{S,RMS}$, $\ddot{Z}_{S,RMS}$) are defined as follows [19]:

$$Z_{S,RMS} = \frac{Z_S}{\sqrt{2}}, \quad \dot{Z}_{S,RMS} = \frac{2\pi f_S Z_S}{\sqrt{2}}, \quad \ddot{Z}_{S,RMS} = \frac{(2\pi f_S)^2 Z_S}{\sqrt{2}} \quad (26)$$

To estimate the expected damage, a simplified assumption is introduced: the SoR signal is treated as a stationary process to estimate the peak rate v_{pSoR} . Assuming statistical independence between the components, the peak rate is expressed as:

$$v_{pSoR} = \frac{1}{2\pi} \sqrt{\frac{\sigma_{Z,r}^2 + Z_{S,RMS}^2}{\sigma_{\dot{Z},r}^2 + \dot{Z}_{S,RMS}^2}} \quad (27)$$

The PDF of the peaks Z_p follows the Rice distribution and can be obtained by integrating Eqn. (21). A closed-form solution exists exclusively for a single tone, yielding

$$p_{Z_p}(Z_p) = \frac{Z_p}{\sigma_{Z,r}^2} e^{-\frac{Z_p^2 + Z_S^2}{2\sigma_{Z,r}^2}} I_0\left(\frac{Z_S Z_p}{\sigma_{Z,r}^2}\right) \quad (28)$$

where I_0 is the modified Bessel function of the first kind of order zero. The expected damage $E[D]$ is then calculated using Eqn. (10):

$$E[D] = \frac{K^m v_{pSoR} T}{C} \int_0^\infty Z_p^m p_{Z_p}(Z_p) dZ_p \quad (29)$$

For a single sinusoid, the integral yields a closed-form solution:

$$E[D] = \frac{K^m v_{pSoR} T}{C} (\sqrt{2}\sigma_{Z,r})^m \Gamma\left(1 + \frac{m}{2}\right) {}_1F_1\left(-\frac{m}{2}, 1, -\alpha_0^2\right) \quad (30)$$

where $\alpha_0 = Z_{S,RMS} / \sigma_{Z,r}$ is the ratio between sinusoidal and random responses, and ${}_1F_1$ is the hypergeometric function. Repeating this procedure for a set of SDOF systems, with increasing natural frequency and constant damping, the expected damage $E[D]$ evaluated in Eqn. (30) can be plotted as a function of the natural frequency, thereby yielding the fatigue damage spectrum.

FDS for Signals with Decoupled Sinusoids. When the harmonic components are spectrally distant and do not simultaneously excite the same SDOF response bandwidth, the resulting FDS can be approximated by the envelope of the damage contributions from individual tones. In this "decoupled" scenario, it is assumed that each system f_0 is primarily excited by only one harmonic or the random background at a time, allowing the application of the single-tone formulation described above for each respective frequency region.

FDS for Signals with Closely Spaced Sinusoids. If multiple sinusoids are closely spaced in frequency, a more general statistical approach is required. For N_b sinusoids with amplitudes a_i and frequencies $f_{S,i}$, superimposed on a random background, the PDF of the envelope (under NB assumptions) is given by Rice's PDF [10]:

$$p_{Z_p}(Z_p) = Z_p \int_0^\infty x J_0(Z_p x) e^{-\frac{\sigma_{Z,r}^2 \cdot x^2}{2}} \prod_{i=1}^{N_b} J_0(Z_{S,i} x) dx \quad (31)$$



where J_0 is the Bessel function of the first kind. The expected peak rate is estimated by extending the previous approximation of Eqn. (27):

$$v_{pSoR} = \frac{1}{2\pi} \sqrt{\frac{\sigma_{Z,r}^2 + \sum_{i=1}^{N_b} Z_{S,i,RMS}^2}{\sigma_{\dot{Z},r}^2 + \sum_{i=1}^{N_b} \dot{Z}_{S,i,RMS}^2}} \quad (32)$$

Since Eqn. (31) cannot be solved in closed form, the expected damage must be evaluated numerically:

$$E[D] = \frac{K^m v_{pSoR} T}{C} \int_0^\infty Z_p^{m+1} \int_0^\infty x J_0(Z_p x) e^{-\frac{\sigma_{Z,r}^2 x^2}{2}} \prod_{i=1}^{N_b} J_0(Z_{S,i} x) dx dZ_p \quad (33)$$

Evaluating the expected damage according to Eqn. (33) for a set of single-degree-of-freedom systems with increasing natural frequencies and constant damping allows the derivation of the FDS. However, the required numerical integration can be computationally demanding, although it is essential for accurately capturing the interaction between closely spaced harmonic components and the random background.

In the following sections, the equivalent PSD approaches are presented.

FDS Inversion with White Noise Approximation. This method synthesizes an equivalent PSD by directly inverting the FDS. This technique has been documented by Halfpenny [26]. Based on the white noise approximation, which assumes the damage at a natural frequency f_0 depends solely on the local PSD value, this approach inverts Eqn. (14), leading to the following equivalent PSD:

$$G_{SoR}(f) = \frac{2(2\pi f)^3}{QK^2} \left[\frac{C \cdot FDS(f)}{f \Gamma\left(1 + \frac{m}{2}\right)} \right]^{\frac{2}{m}} \quad (34)$$

where, instead of an FDS derived from random signals, a SoR FDS, evaluated in the time domain or in the frequency domain is used.

The result is a continuous PSD profile that approximates the damage potential of the original SoR signal across the entire frequency spectrum. The resulting outcome is contingent upon the signal duration T , the quality factor Q , and the fatigue parameters of the material C and m .

This approach provides a computationally immediate profile, though, as Fig. 5 shows, the approximation tends to distribute energy over adjacent frequencies, resulting in less pronounced peaks compared to other methods.

Lalanne's Iterative Method. Lalanne proposes a recursive refinement process for generating a random PSD that matches a target FDS derived from a SoR specification (in time or frequency domain) [12,31]. The starting point for this process is an initial guess, $G_0(f)$, which can be defined as constant on all frequencies or in a partial frequency range. The *random* damage spectrum FDS_0 is then derived from $G_0(f)$, using Eqn. (12). The PSD is then corrected through subsequent iterations according to the following expression:

$$G_{i+1}(f) = G_i(f) \left(\frac{FDS_{target}(f)}{FDS_i(f)} \right)^{\frac{2}{m}} \quad i = 0, 1, \dots, \text{num. of iterations} \quad (35)$$

where $FDS_{target}(f)$ is the SoR derived fatigue damage spectrum. This method has been demonstrated to converge rapidly and ensure superior peak resolution in comparison to direct inversion of the FDS. The efficacy of this method is contingent upon the initial guess and the exponent m of Basquin's law.



Furthermore, the iterative synthesis process explicitly accounts for the spectral shape of the response during the FDS calculation. The RMS of the response is computed through exact spectral integration, thereby avoiding the simplifications associated with the white noise input approximation. Consequently, the energy required to achieve damage equivalence with the original sinusoidal components remains strictly confined within narrow frequency bands, ensuring that the power is not erroneously distributed to neighbouring frequencies, as illustrated in Fig. 5.

RMS Equivalence Method. The RMS equivalence method, described by Cho [16], preserves the total power of the signal. A sinusoidal component of amplitude a is converted into an equivalent PSD distributed over a frequency range Δf centered around the sine frequency. The total root mean square value is:

$$\sigma_{SoR} = \sqrt{\sigma_x^2 + \frac{a^2}{2}} \quad (36)$$

where σ_x is the RMS of the random component, and a is the amplitude of the sinusoidal component. Denoting with $G_r(f)$ the PSD of the signal's random Gaussian component, the resulting equivalent PSD, $G_{SoR}(f)$, is defined as:

$$G_{SoR}(f) = \begin{cases} G_r(f) + \frac{a^2}{\Delta f} \left(1 - \frac{2|f - f_s|}{\Delta f} \right) & |f - f_s| \leq \frac{\Delta f}{2} \\ G_r(f) & \text{elsewhere} \end{cases} \quad (37)$$

The selection of the frequency bandwidth, denoted by Δf , for the equivalent PSD is arbitrary and requires careful engineering judgment. To prevent the occurrence of sampling artifacts, it is imperative that the quantity known as Δf not be reduced below the frequency resolution (df) of the PSD. Conversely, an excessively wide Δf should be avoided to prevent spectral leakage, which would distribute the harmonic power across frequencies unrelated to the original deterministic excitation.

Irvine's Narrowband PSD Conversion Method. A different technique within the practical framework is the conversion method proposed by Tom Irvine [14]. This approach aims to replace the deterministic sinusoidal component with an equivalent NB random PSD, typically centred at the sine frequency f_s . Unlike the RMS equivalence, Irvine's method is specifically designed to ensure that the synthetic random vibration produces the same fatigue damage as the original harmonic tone when applied to an SDOF system.

The equivalent PSD is characterized by a "chimney" geometry (Fig. 5), where the power is concentrated within a narrow frequency band Δf .

The logic for converting each sinusoid to an equivalent NB random signal is based on two primary criteria:

- **Cycle Threshold Criterion:** This method ensures that the NB random signal contains a specific number of cycles whose amplitudes exceed the peak response of the original sine wave, $a_{resp} \approx Q \cdot a$, where Q is the quality factor and a is the amplitude of the sine wave. Since the peaks of a NB random process follow a Rayleigh distribution, this criterion is often considered a conservative boundary for structural integrity.
- **Pseudo-damage Equivalence Criterion:** This is the most widely adopted version of the method. It defines the intensity of the NB PSD by equating the calculated "pseudo-damage" of the random process to that of the sine tone. For a sinusoidal input of amplitude a , frequency f_s , and duration T , the damage D_{sine} is:

$$D_{sine} = f_s \cdot T \cdot (Q \cdot a)^m \quad (37)$$

where m is the fatigue exponent. An initial NB PSD equivalent level $G_{NB \text{ start}}$ is arbitrarily selected and its damage D_{start} calculated via the Rayleigh or Dirlik models. The final equivalent PSD (G_{NB}) level is evaluated by means of a correction factor which accounts for the sinusoidal (D_{sine}) and initial NB (D_{start}) damage values. More precisely, the correction formula becomes:

$$G_{NB} = G_{NB \text{ start}} \left(\frac{D_{\text{fine}}}{D_{\text{start}}} \right)^{\frac{2}{m}} \quad (38)$$

This ensures the equivalence of fatigue damage values between the sinusoid and the NB random PSD.

For both methodologies, the definition of the equivalent PSD bandwidth is a critical parameter. In the absence of strict regulatory constraints, Irvine proposes a pragmatic approach by fixing the frequency interval to one-twelfth of an octave ($1/12 \text{ oct}$) centred at the frequency of the sinusoidal component. It must be noted that the selection of a different bandwidth invariably yields distinct solutions with respect to input PSD levels, even though all such configurations are derived to be equi-damaging in relation to the original sine wave.

Equivalent PSDs comparison and considerations. An example of a MIL-STD-810 SoR profile is visible in Fig. 4. A random PSD, representing the Gaussian component, is shown in blue and refers to the left axis, while the pure tones are depicted in red and refer to the right axis. Since dimensionally they represent different physical quantities, it is impossible to unify these two types of information in the same graph.

Based on the SoR specification described above, Fig. 5 compares the equivalent PSDs generated by each method.

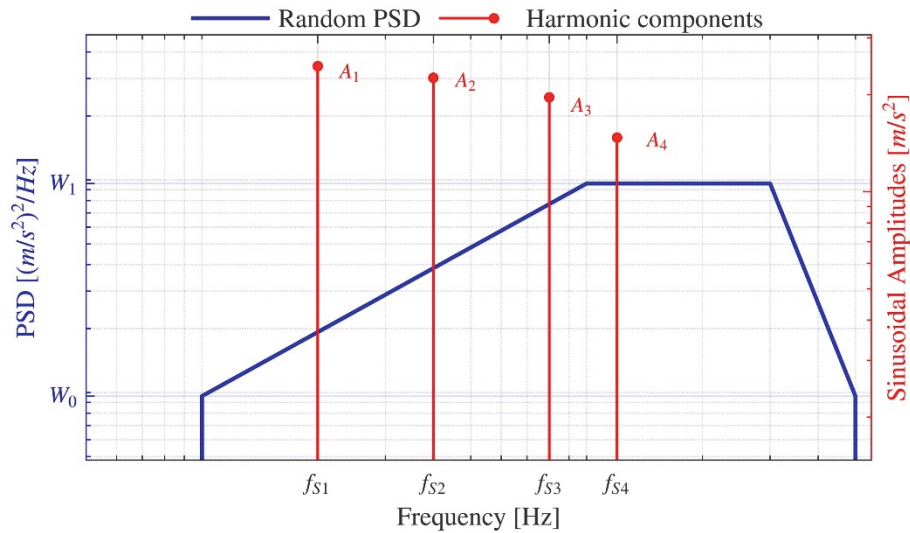


Figure 4: Sine-on-Random MIL-STD-810 profile.

Despite the preservation of signal power or damage potential, the transition from a deterministic to a stochastic representation introduces significant discrepancies in the peak value statistics. A Gaussian signal derived, for instance, from an RMS equivalent PSD typically exhibits much higher peak-to-RMS ratios than a pure sinusoid. Specifically, while a sinusoidal tone of amplitude a generates cycles of constant amplitude a , its NB Gaussian equivalent generates a cycle distribution that follows a Rayleigh PDF. In the latter case, the non-zero probability of high-amplitude peaks in the distribution tails leads to a severe overestimation of fatigue damage. This effect is particularly pronounced for materials with high Basquin exponents (m), as damage is exponentially sensitive to peak stress levels.

Among the equivalent approaches, the RMS Equivalence and the Irvine methods strictly depend on the sine-to-random conversion bandwidth, which must be chosen with judgment.

The RMS Equivalence method is the only approach independent of the Basquin S-N curve slope (m), as it aims for energy equivalence rather than fatigue damage equivalence.

For Lalanne's and White Noise Approximation methods, information regarding the S-N curve is required: the intercept C is necessary for the FDS evaluation (it can occasionally be set as unity), while the quality factor Q must be chosen based on the specific scenario. As previously established, while the White Noise approach is straightforward to implement but yields approximate results, Lalanne's method is slightly more complex yet provides higher accuracy and better reflects physical reality.

Fig. 6 provides a schematic classification of the previously described approaches, highlighting their similarities and differences.

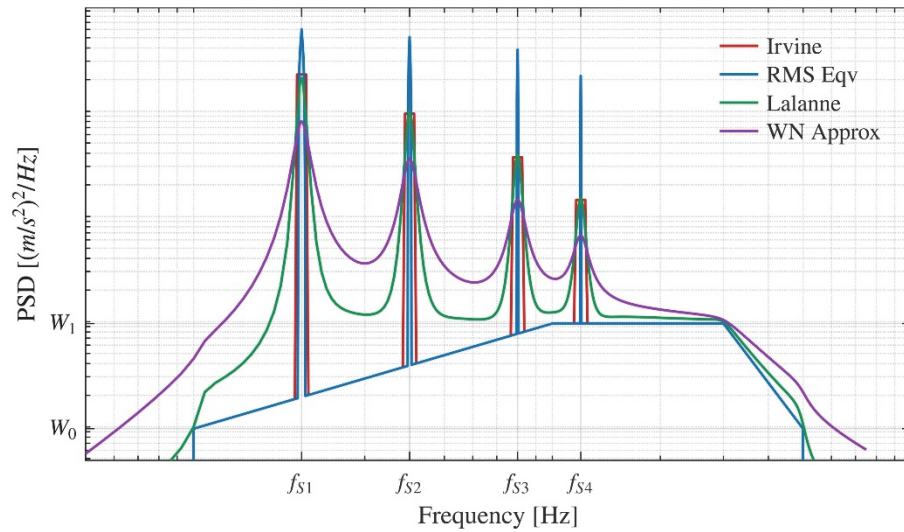


Figure 5: Comparison between Equivalent PSDs.

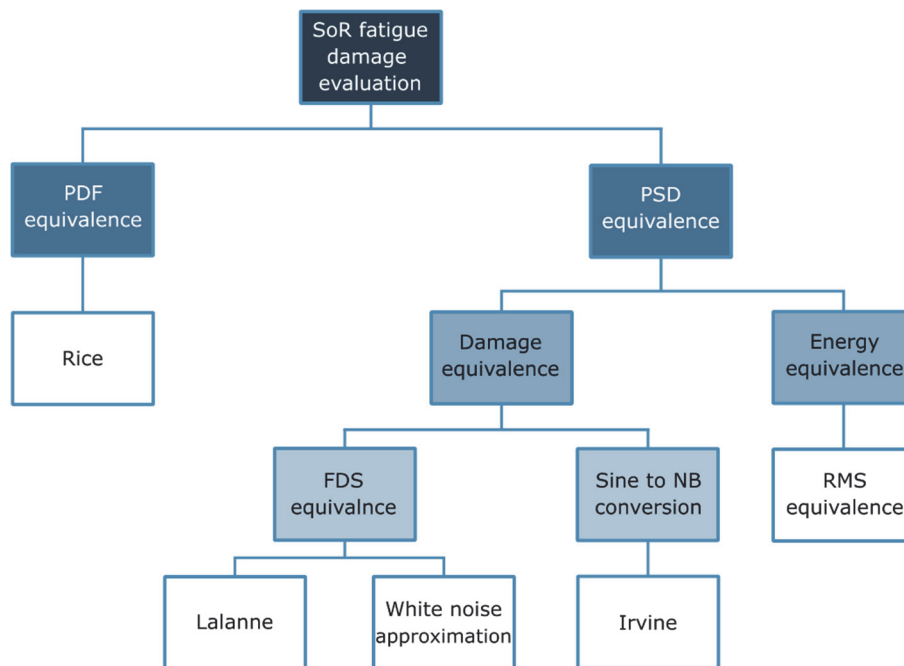


Figure 6: Graphic summary of the SoR fatigue assessment methods.

NUMERICAL COMPARISON

This section presents a comparative numerical validation of the SoR fatigue assessment approaches previously introduced. The objective is twofold: to quantify their predictive accuracy and to map their operational limits based on the spectral characteristics of the excitation.

The time-domain damage calculation – utilizing synthetic time histories, rainflow counting, and the Palmgren-Miner rule – serves as the exact benchmark. Despite its high computational cost, the RFC approach is adopted as the baseline because it avoids the statistical approximations inherent in spectral methods.



To isolate the impact of the spectral characteristics on damage accumulation, the input acceleration specifications are assumed to represent stress excitations directly. Consequently, the damage is computed as a pseudo-damage index ($C=1$), effectively eliminating the need for a conversion factor between the acceleration input and the resulting stress. This simplification is adopted as the study focuses on a comparative validation of the methods rather than the absolute life prediction of a physical component. Furthermore, this specific assessment does not model a vibrating system. Since no relative displacement calculation is performed, the proportionality factor K is omitted from the pseudo-damage formulation.

In this comparative analysis, a quality factor value of $Q=10$, common for steel structures, was assumed for the dynamic behaviour of the SDOF systems that are required to apply all the PSD equivalent methods, except for the RMS equivalence method.

Moreover, a value of $m=3$, common for welded joints in steel structures, was selected for the pseudo damage evaluations according to the Eurocode 3: Design of steel structures - Part 1-9: Fatigue standard. Hence, the results and conclusions discussed in this manuscript are to be considered exclusively in relation to the selected values of Q and m . Different values of Q and m may in facts yield different pseudo damage estimates and the methods' performances could be different.

The analysis is structured incrementally across three complexity levels:

- **Narrowband scenario:** A single sine tone superimposed on an NB random process.
- **Wideband scenario:** A single sine tone on a WB random background.
- **Multi-Sine (MIL-STD) Scenario:** Application to a standard qualification profile with multiple tones.

For each scenario, performance is quantified via the percentage deviation of the expected damage from the time-domain benchmark, evaluating the trade-off between accuracy and computational cost. The statistical representativeness of the models is rigorously verified by comparing the PDFs of the cycle amplitudes (s) and their corresponding cumulative load spectra.

Analysis Plan. Cycle counting and damage evaluation are made directly on the input SoR profiles, which are later described, and the time-domain benchmark is obtained through 3600 s synthetic time-histories. The evaluation framework is twofold, assessing both the percentage error on the expected damage $E[D]$ and the statistical representativeness of the cycle distributions. Calculations assume an arbitrary S-N curve ($m=3$, $C=1$), yielding a relative Damage Index where the scaling effect of C cancels out.

Beyond standard stress amplitude PDFs $p_s(s)$ and cumulative load spectra, the statistical analysis rigorously examines the damage integrand function:

$$p_s(s) \cdot s^m \quad (40)$$

This function is the integrand of Miner's linear damage formula, expressed in Eqn. (10).

The area under this curve dictates the Damage Index. Plotting this integrand is crucial, as it visually isolates the amplitude bands that govern fatigue accumulation, offering a stricter test of statistical fidelity than raw PDFs.

The probability distributions and load spectra resulting from the equivalent random profiles, as well as the pseudo damage indexes, were obtained via Dirlik's PDF method, maintaining the frequency domain approach to damage evaluation, instead of extracting time histories from the PSDs and counting the stress cycles via the RFC algorithm.

Single sine on NB random

This scenario features a Narrowband random background defined between 475 Hz and 525 Hz, centred at $f_r = 500$ Hz. The resulting random stress RMS value is $\sigma_{s,r} \approx 2.24$ MPa. A deterministic sine wave $y(t) = a \sin(2\pi f_s t)$ is superimposed on this stochastic process $x(t)$. The experimental design investigates the influence of two fundamental dimensionless parameters:

- **Amplitude Ratio** $a / \sigma_{s,r}$: Quantifies the dominance of the deterministic component. For this analysis, a value of $a / \sigma_{s,r} = 4$ has been used.
- **Frequency Ratio** $k = f_s / f_r$: Defines the spectral position of the tone relative to the random band. The analysis covers out-of-band low-frequency tones ($k=0.05$) and an in-band resonance case ($k=1$).

Fig. 7 summarizes the two load conditions in this scenario: A NB random noise and a single pure tone, whose frequency ratio varies between the values $k=0.05$ and $k=1$.

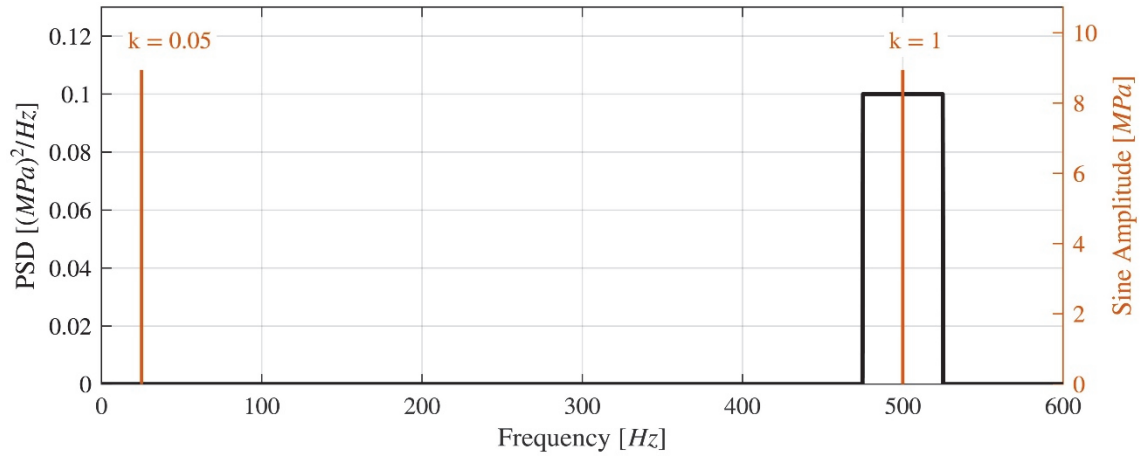


Figure 7: Narrow-Band Random Background $\gamma = 1$ ($\epsilon = 0.06$) - Frequency Ratios $k = 0.05$ and $k = 1$.

Results and Discussions. Figs. 8 and 9 show the results for the spectral separation case study. Due to the wide spectral separation ($k=0.05$), there is no dynamic interaction between the two signals; the random noise effectively rides on the harmonic component, which acts as a slowly varying mean stress. As a result, the reference RFC PDF exhibits a distinctly bimodal distribution: a primary concentration at lower amplitudes representing the noise-induced cycles, and a secondary, marked tail capturing the harmonic contribution.

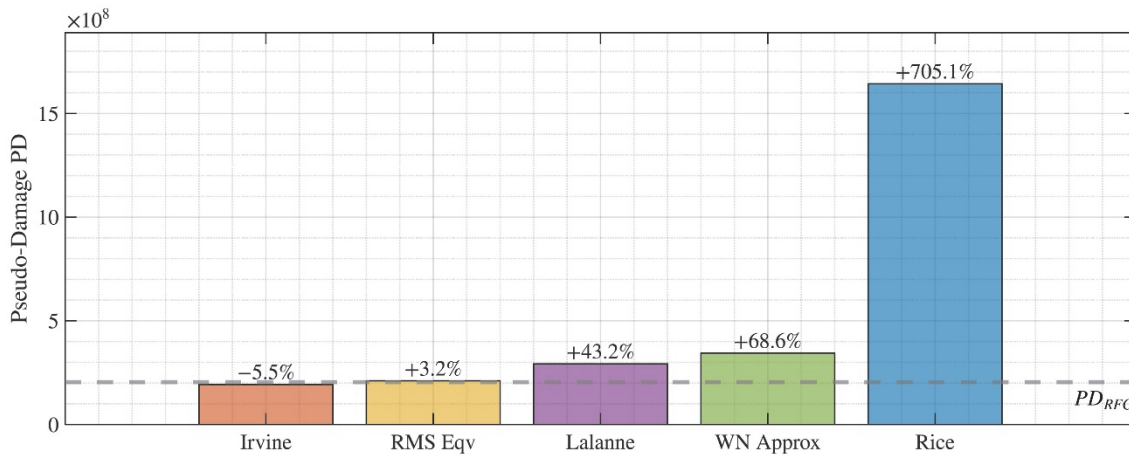


Figure 8: Comparison of pseudo-damage estimates under SoR loading with a narrow-band background ($k = 0.05$). The dashed grey line represents the reference rainflow counting pseudo-damage (PD_{RFC}). Percentages above the bars indicate the relative error of each predictive method.

An analysis of the predictive models yields the following critical observations:

- **Rice's Method:** In the selected scenario, the model yields a heavily conservative damage estimate, registering an error of approximately +700%. The analysis of the results seems to show that this severe overestimation stems directly from the violation of the model's fundamental assumptions. The analytical PDF incorrectly collapses almost entirely around the sine amplitude level (≈ 9 MPa), completely failing to capture the dense concentration of low-amplitude cycles present in the RFC distribution. This deviation propagates into the damage integrand, yielding an area under the curve significantly larger than the actual one, which leads to the excessively conservative damage prediction. However, the model's cumulative load spectrum does converge towards the actual reference curve strictly in the extreme high-amplitude (low-cycle) region.

- The approaches based on Equivalent PSD show a fundamental inability to represent the actual physics of the SoR phenomenon, mainly visible in the integrand function panel (top right). While they exhibit a reasonable match in the low-amplitude/high-cycle region, their probability tails seem to fail to account for the presence of a pure tone. This structural flaw leads to an overestimation of high-amplitude events and a severe penalization of medium-amplitude cycles, exactly where the influence of the sine wave is dominant. From this behaviour, one can infer that the relatively low errors in damage estimation obtained from the analysis of this loading scenario come from a mathematical compensation of the integrands' areas, combined with a relatively low Basquin exponent (m), rather than actual physical accuracy. The Irvine and RMS Equivalence methods seem to yield the best estimates, suggesting being highly useful in practice despite being physically inaccurate. Lalanne and White Noise Approximation methods are too conservative in this scenario.

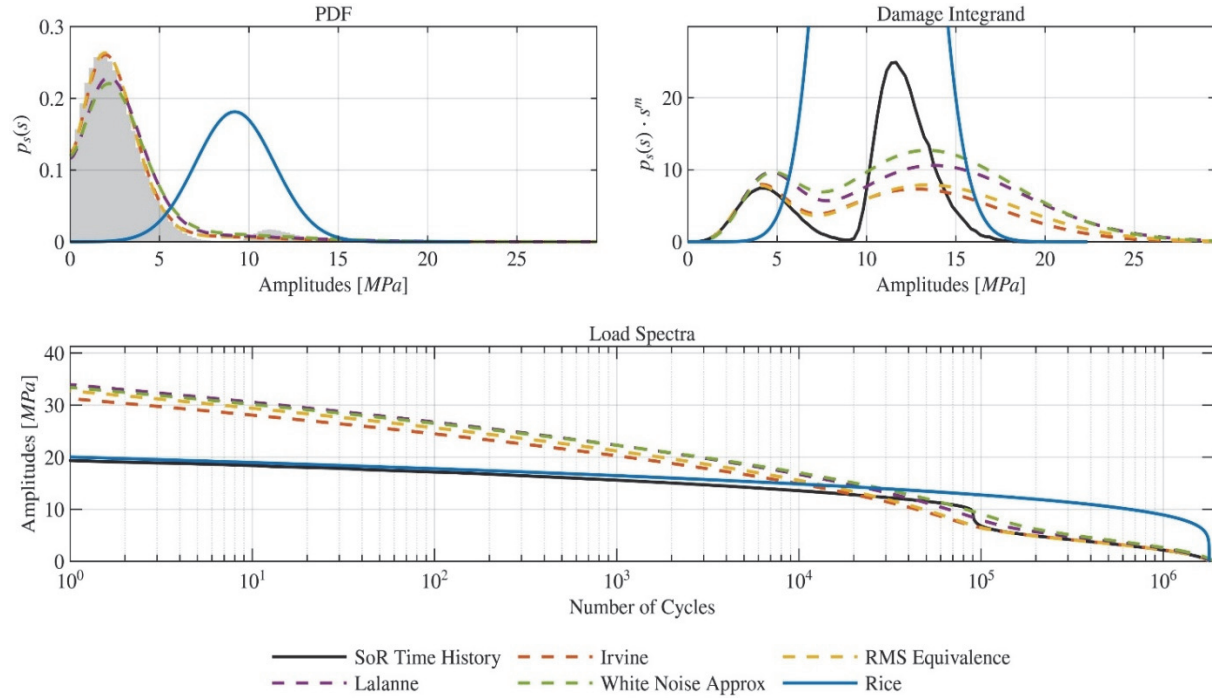


Figure 9: Statistical comparison of the predictive models under SoR loading with a narrow-band background ($\kappa = 0.05$). The reference RFC probability density function is depicted as a shaded grey histogram in the first panel.

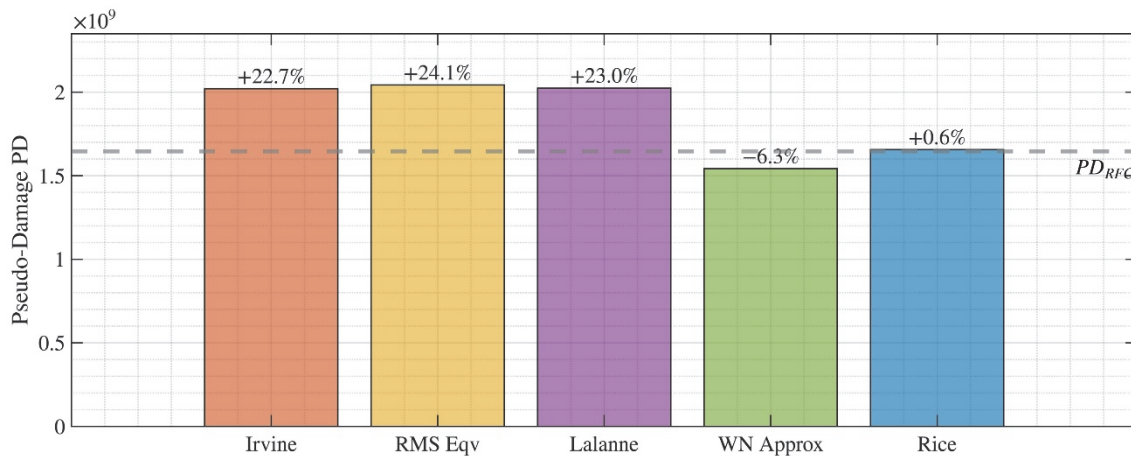


Figure 10: Comparison of pseudo-damage estimates under SoR loading with a narrow-band background ($\kappa = 1$). The dashed grey line represents the reference rainflow counting pseudo-damage (PD_{RFC}). Percentages above the bars indicate the relative error of each predictive method.

Figs. 10 and 11 represent the scenario where the sine wave frequency perfectly matches the mean frequency of the random background ($k=1.0$). This condition falls strictly within the validity domain of Rice's hypothesis.

An analysis of the results yields the following critical observations:

- **Rice's Method:** Operating fully within its theoretical assumptions, this model proves highly accurate. The analytical PDF and the damage integrand perfectly overlap the corresponding RFC-derived functions. Consequently, both the damage integrand and the cumulative load spectrum exhibit excellent agreement with the reference, resulting in a negligible damage estimation error ($\approx 0.6\%$).
- **White Noise Approximation:** This approach exhibits the poorest overall performance. It completely fails to track the reference statistics and uniquely results in an underestimation of the pseudo-damage.
- **Equivalent PSD approaches:** The Irvine, Lalanne, and RMS Equivalence methods exhibit nearly identical behaviours, producing load spectra typical of purely random signals rather than SoR loadings. An initial lack of fidelity in modelling the actual PDF results in integrand curves that exhibit higher values than the reference at high amplitude cycles, and lower values elsewhere. As observed in the previous case, their seemingly acceptable damage estimation errors seem to be mathematical artifacts resulting from the cancellation of areas under the integrand curve.

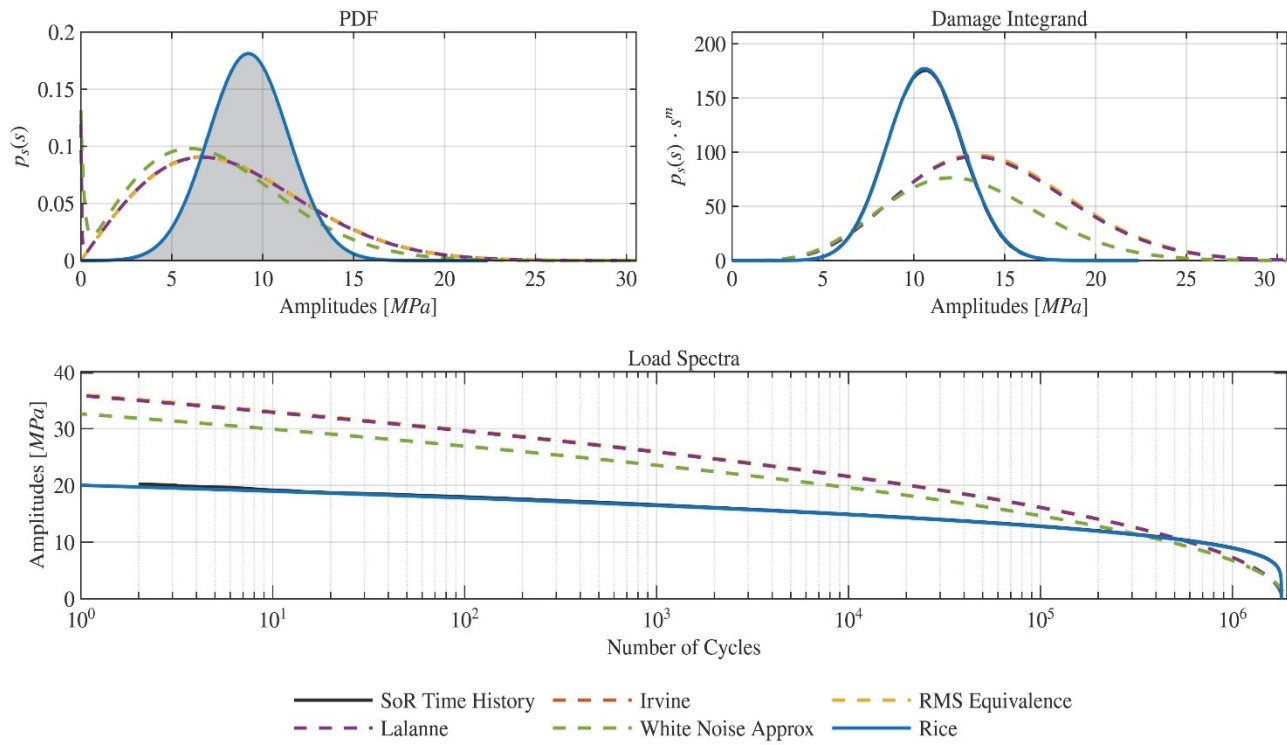


Figure 11: Statistical comparison of the predictive models under SoR loading with a narrow-band background ($k = 1$). The reference RFC probability density function is depicted as a shaded grey histogram in the first panel.

Single sine on WB random

The second analysed scenario features a loading condition comprising a wide-band random background, defined between 10 Hz and 500 Hz, with a mean frequency $f_r = 255$ Hz (depicted in black in Fig. 12). Consequently, the resulting RMS value of the noise is $\sigma_{s,r} = 7$ MPa. A pure sinusoidal tone $y(t) = a \sin(2\pi f_s t)$ is superimposed onto the random signal $x(t)$. The test plan for this scenario employs the same set of dimensionless parameters used in the previous analysis ($a / \sigma_{s,r} = 4$ and $k = (0.05, 1)$).

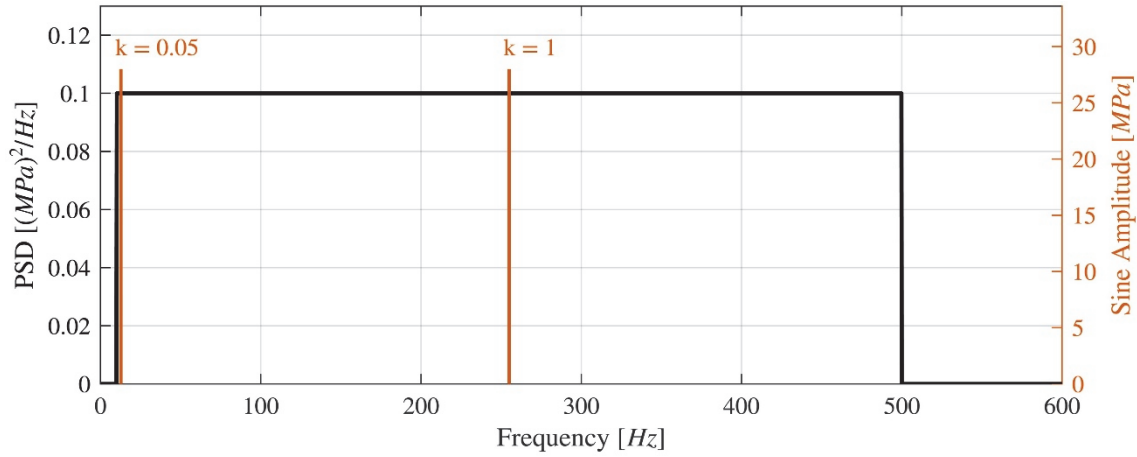


Figure 12: Wide-Band Random Background $\gamma = 1$ ($\epsilon = 0.66$) - Frequency Ratios $k = 0.05$ and $k = 1$.

Results and Discussions. Figs. 13 and 14 display the results obtained from the WB random background analysis with spectral separation ($k=0.05$).

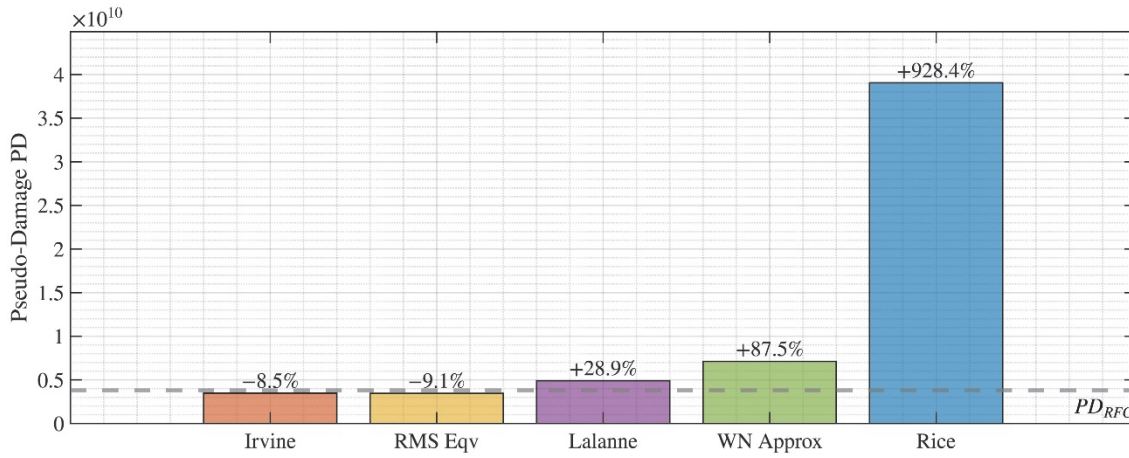


Figure 13: Comparison of pseudo-damage estimates under SoR loading with a wide-band background ($k = 0.05$). The dashed grey line represents the reference rainflow counting pseudo-damage (PD_{RFC}). Percentages above the bars indicate the relative error of each predictive method.

The analysis of the results brings the following observations:

- Rice's method accurately tracks the Time History load spectrum in the high-amplitude, low-cycle regime; however, it severely overestimates the medium-amplitude range, where cycle counts are significantly higher. Consequently, the resulting damage estimation is extremely conservative (more than 10 times higher than the RFC pseudo damage value), confirming the trend already observed in the Narrowband scenarios.
- In this scenario, Equivalent PSD approaches exhibit low damage estimation errors. However, as observed previously, this apparent accuracy is merely the result of an area compensation artifact within the damage integrand. Specifically, the Irvine and RMS Equivalence methods yield reasonable estimates, limiting the underestimation to within 10%. Lalanne's method proves more conservative (+29%), whereas the White Noise Approximation apparently fails to capture the underlying physics, overestimating the damage by nearly 88%. This severe discrepancy is a direct consequence of the white noise assumption inherent to its mathematical formulation.

The last presented scenario, described in Figs. 15 and 16, is characterized by an amplitude ratio $a / \sigma_{s,r} = 4$ and spectral coincidence between the components ($k=1$).

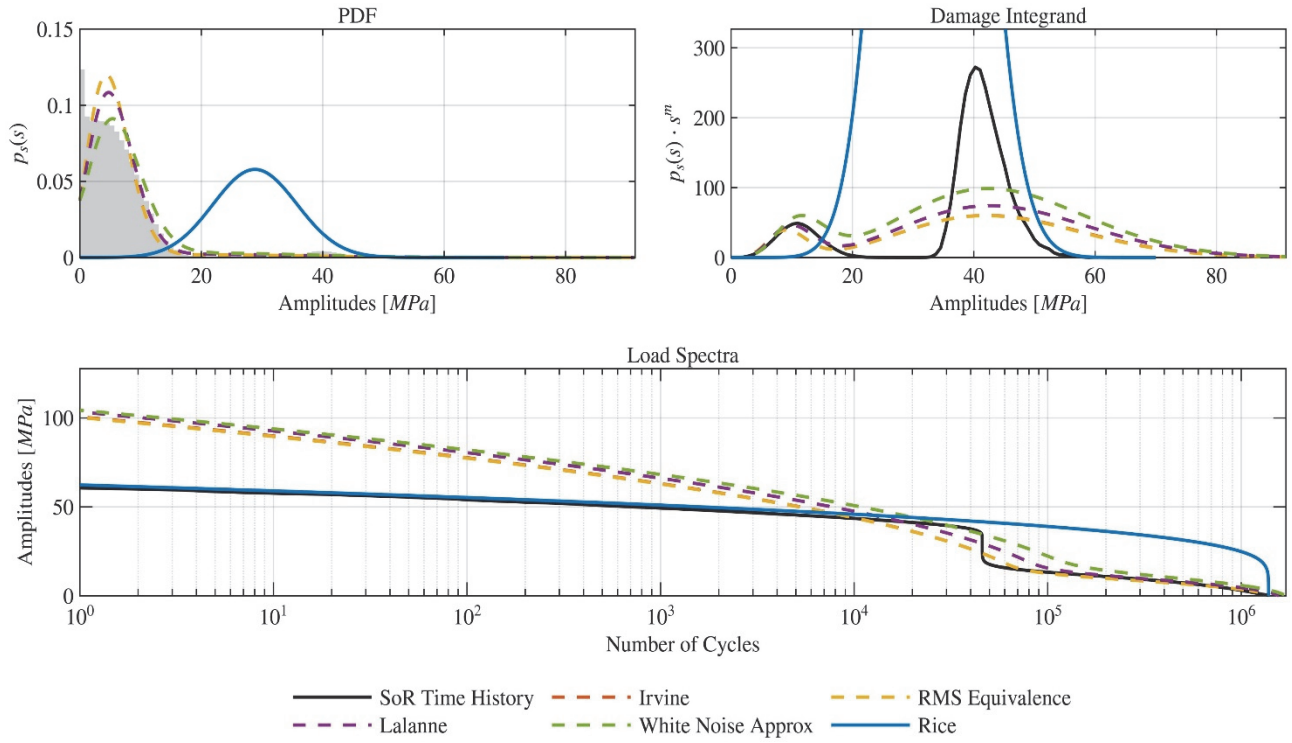


Figure 14: Statistical comparison of the predictive models under SoR loading with a wide-band background ($k = 0.05$). The reference RFC probability density function is depicted as a shaded grey histogram in the first panel.

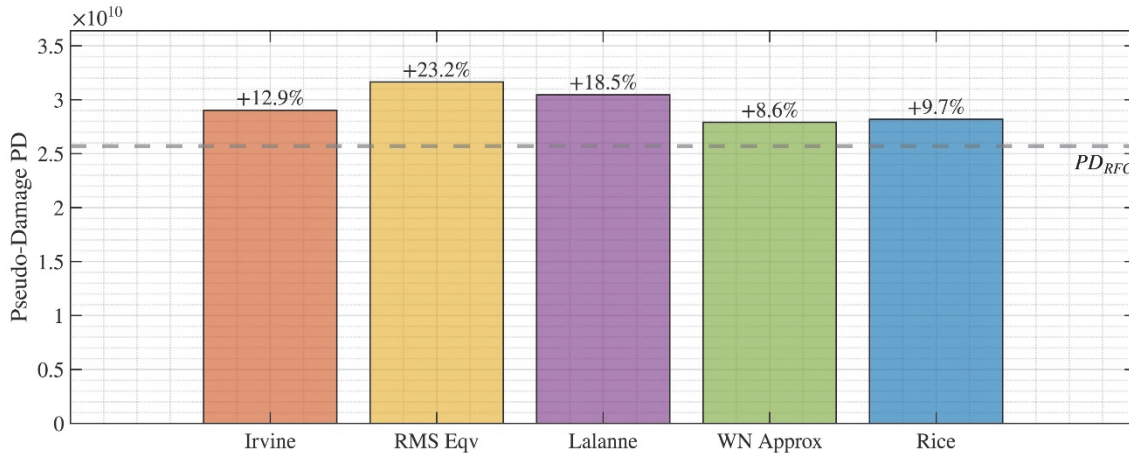


Figure 15: Comparison of pseudo-damage estimates under SoR loading with a wide-band background ($k = 1$). The dashed grey line represents the reference rainflow counting pseudo-damage (PD_{RFC}). Percentages above the bars indicate the relative error of each predictive method.

The comparative analysis results in the following considerations:

- The evidence suggests that even in the presence of wide-band noise, when spectral coincidence occurs between the harmonic and stochastic components, Rice's approach emerges as the most accurate overall. It overestimates the pseudo-damage by merely 10%, showing excellent adherence to the reference PDF, cumulative load spectrum, and damage integrant curves.
- The Equivalent PSD approaches yield conservative damage estimates, with errors of 13% for Irvine, 22% for RMS Eqv, 18.6% for Lalanne, and 8.6% for the WN Approximation. Despite the numerical proximity of the final Damage Index, none of these methods accurately reproduces the morphology of the true Load Spectrum. Instead, they predict an excessive number of high-amplitude cycles that are entirely absent from the original signal.

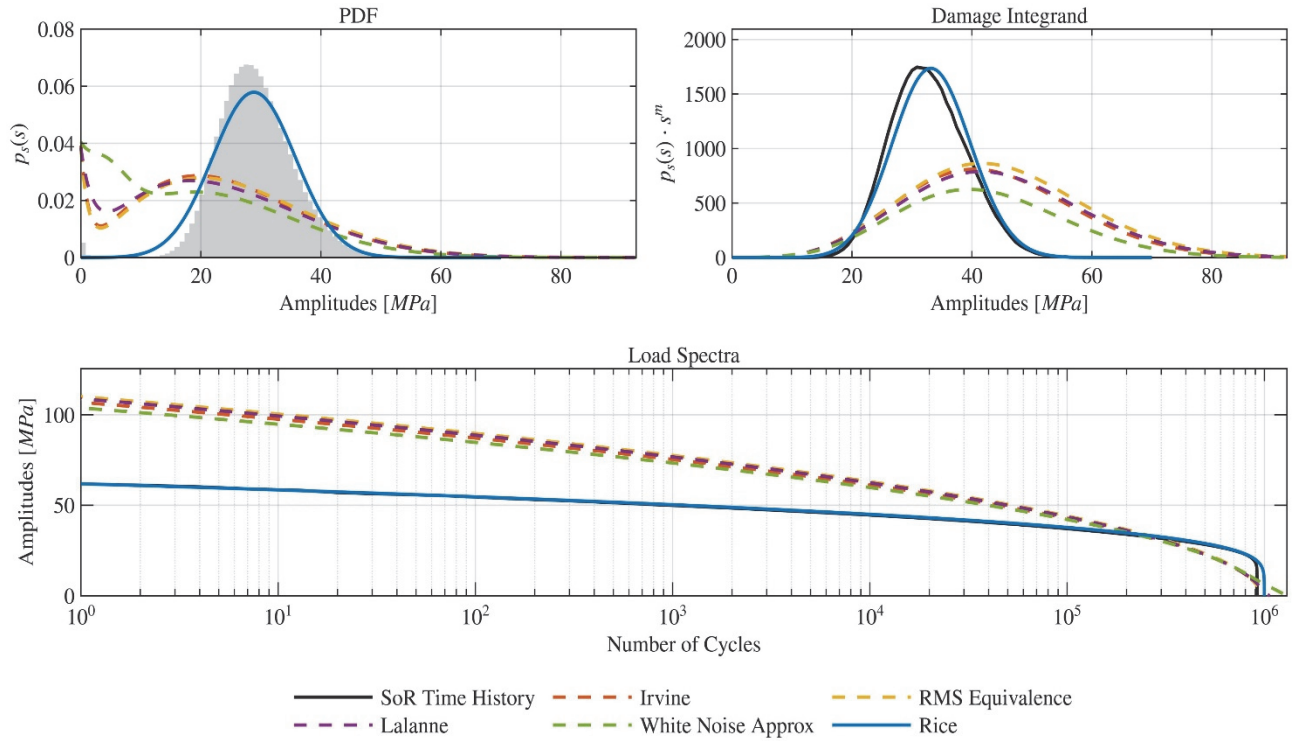


Figure 16: Statistical comparison of the predictive models under SoR loading with a wide-band background ($k = 1$). The reference RFC probability density function is depicted as a shaded grey histogram in the first panel.

Cross-case observations. The comprehensive analysis reveals that, as the bandwidth of the random component increases, Rice's PDF exhibits excellent fidelity when $f_s \approx f_r$, leading to highly accurate estimates. Conversely, under conditions of spectral separation, this method proves strictly conservative, consistently yielding positive errors that can reach up to 900%. This behaviour stems directly from the assumption that every local maximum belongs to the upper envelope, thereby ignoring the presence of small-amplitude cycles nested within larger ones – a dominant phenomenon in wideband signals. Consequently, the results suggest that while Rice's formulation represents a safe approach for preliminary sizing, its application for the structural optimisation of components subjected to SoR profiles is strongly discouraged, as it invariably leads to severely over-engineered designs. In this context, Equivalent PSD approaches (specifically Irvine, Lalanne, and RMS Eqv) seem to emerge as the most robust engineering solution for Wideband scenarios. Although these methods fail to faithfully replicate the bimodal morphology of the amplitude distribution, tending instead to 'smear' the harmonic energy across a fictitious frequency band, they benefit from a highly effective error cancellation mechanism:

- The transformation of the deterministic tone into a random band inherently underestimates the probability density around the sine peak.
- Simultaneously, this mathematical transformation artificially inflates the distribution tails compared to reality.

The net result is that, upon integrating the damage curve, the underestimation at medium amplitudes and the overestimation at extreme amplitudes tend to balance out. This provides final Damage Index estimates that generally fall within an acceptable $\pm 20\%$ accuracy band. The sole exception is the WN Approximation method. By reducing the entire mixed spectrum to pure white noise, it fundamentally fails to capture the energy complexity of the actual signal, proving systematically too severe and entirely unsuitable for these loading profiles.

MIL-STD-810 SoR profile

For this analysis, a representative SoR loading profile was adopted from the MIL-STD-810 standard (specifically, the OH-58A/C helicopter specifications). For comparative purposes, the original acceleration amplitudes were numerically transposed into the stress domain. This was achieved by applying a direct 1:1 mapping (substituting m/s^2 with MPa) to evaluate the fatigue response while preserving the exact spectral morphology of the reference signal (Fig. 17).

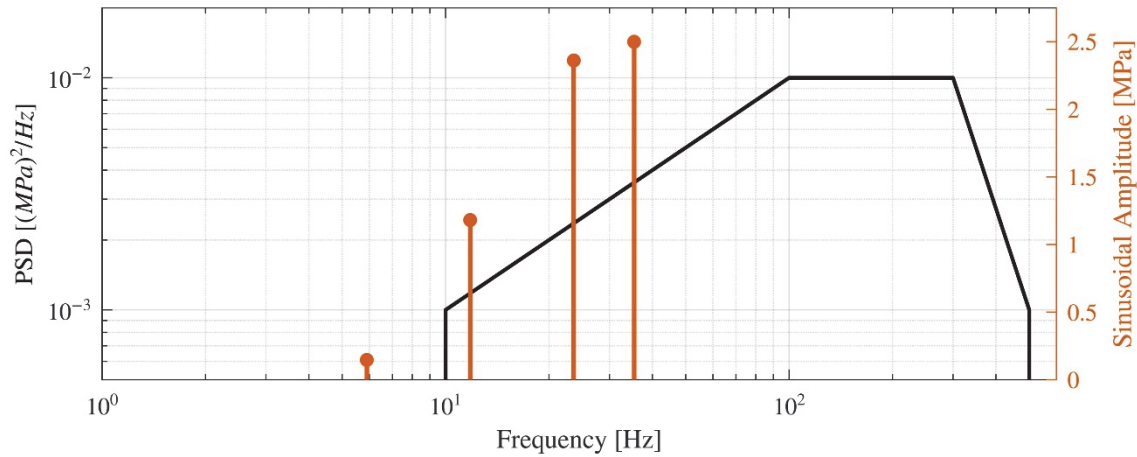


Figure 17: MIL-STD-810 SoR profile. Helicopter model OH-58A/C. Random RMS value $\sigma = 1.8$ MPa. Harmonic frequencies: 5.9 Hz, 11.8 Hz, 23.6 Hz, 35.4 Hz. Harmonic amplitudes: 0.15 MPa, 1.18 MPa, 2.36 MPa, 2.5 MPa.

The deterministic components correspond to the harmonics generated by the two-bladed main rotor ($n_b = 2$). Given the nominal rotational speed $f_{\text{rot}} \approx 354$ RPM, the excitation frequencies are defined as $f_1 = f_{\text{rot}} = 5.9 \text{ Hz}$, $f_2 = n_b \cdot f_{\text{rot}} = 11.8 \text{ Hz}$, $f_3 = 2n_b \cdot f_{\text{rot}} = 23.6 \text{ Hz}$ and $f_4 = 3n_b \cdot f_{\text{rot}} = 35.4 \text{ Hz}$. For the numerical simulation, the phases of the harmonic components within the SoR signal were set to zero to represent a realistic operational scenario. This configuration induces constructive interference of the peaks, thereby amplifying the local signal amplitude. Furthermore, this condition strictly violates the “incommensurable frequencies” (or random phases) assumption underlying Rice's theoretical model, thus anticipating a discrepancy between the actual peak distribution and the analytical prediction.

Results and Discussions. The analysis of the results presented in Figs. 18 and 19 highlights the following:

- The interaction of the in-phase sinusoidal tones generates a periodic signal, whose waveform causes a series of peaks within the integrand function. This is also evident in the cumulative load curve of the Time History, which does not exhibit a single inflection point (as seen in single-tone cases), but rather displays a complex trend resulting from the dynamic interaction between the periodic signal and the noise. Rice's method, unable to model the specific phase relationship between the harmonics, treats the entire signal as a modulated narrow-band process and calculates its envelope. This is thought to be the reason that leads to the overestimation of the probability of occurrence for high amplitudes and, consequently, yields a consistently conservative damage estimate.
- The approaches based on spectral transformation demonstrate notable accuracy. Although these methods completely lose the waveform information (by flattening the tones into noise bands), they managed to provide engineering-sound damage estimates. As already discussed, this success may be due to the area cancellation effect: the error committed in modelling the exact shape of the PDF is balanced out when integrated over the entire domain, leading to a final Damage Index very close to that of the Time History. The sole exception is the WN Approx method, which proves to be the least accurate due to the oversimplification of the spectrum (white noise assumption).

In conclusion, the application of Equivalent PSD methods to the MIL-STD scenario demonstrated valid accuracy margins (except for the WN Approx method). However, it must be considered that an increase in the Basquin exponent m could amplify the discrepancies between the model and reality, thereby reducing the accuracy of these methods.

Furthermore, spectral equivalence methods present a fundamental issue related to the nature of the signal: substituting pure tones (bounded amplitudes) with Narrowband components (Gaussian processes with infinite tails) fundamentally alters the statistical nature of the excitation. This error is exacerbated in the multi-Sine scenario: replacing four interacting, in-phase deterministic tones with four uncorrelated noise bands means losing the information regarding the constructive interference of the peaks, which can result in even worse damage estimates than those in Fig. 18.

A final observation regarding these methods concerns their inherent formulation: aside from the energy equivalence approach, algorithms such as Lalanne and Irvine are formulated to ensure damage equivalence on the *response* of SDOF systems, meaning systems with non-negligible dynamics. Although they incorporate corrections based on the m coefficient, these methods implicitly assume a dynamic amplification factor (Q) which, if absent or different from the assumed value, can introduce systematic biases into the Damage Index evaluation.

Since most standard SoR specifications feature wideband random noise, Rice's method often operates outside its underlying assumptions, treating the entire process as a narrowband signal and overestimating the Damage Index. Although a cautious estimate is generally desirable, this PDF approach proves to be excessively conservative, rendering the results practically unusable.

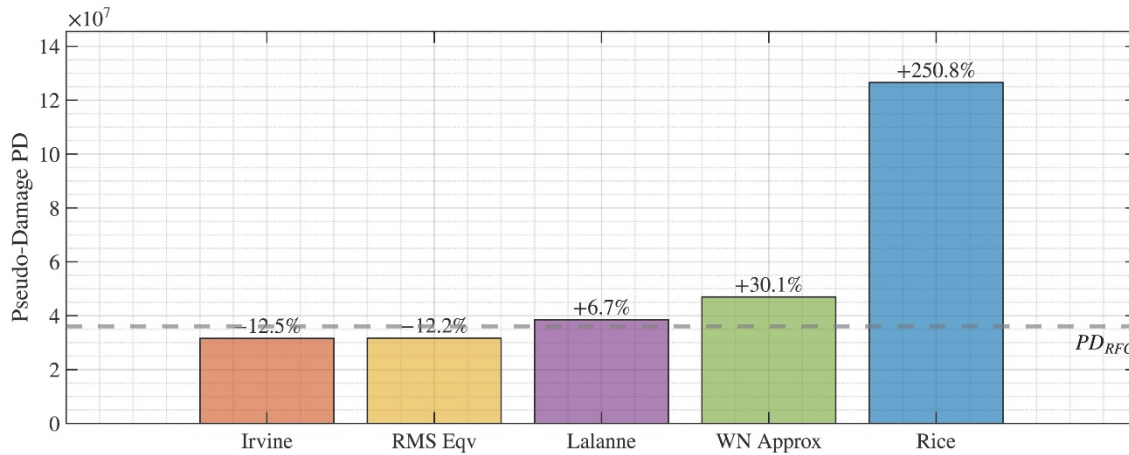


Figure 18: Comparison of pseudo-damage estimates under MIL-STD 810 SoR loading. The dashed grey line represents the reference rainflow counting pseudo-damage (PD_{RFC}). Percentages above the bars indicate the relative error of each predictive method.

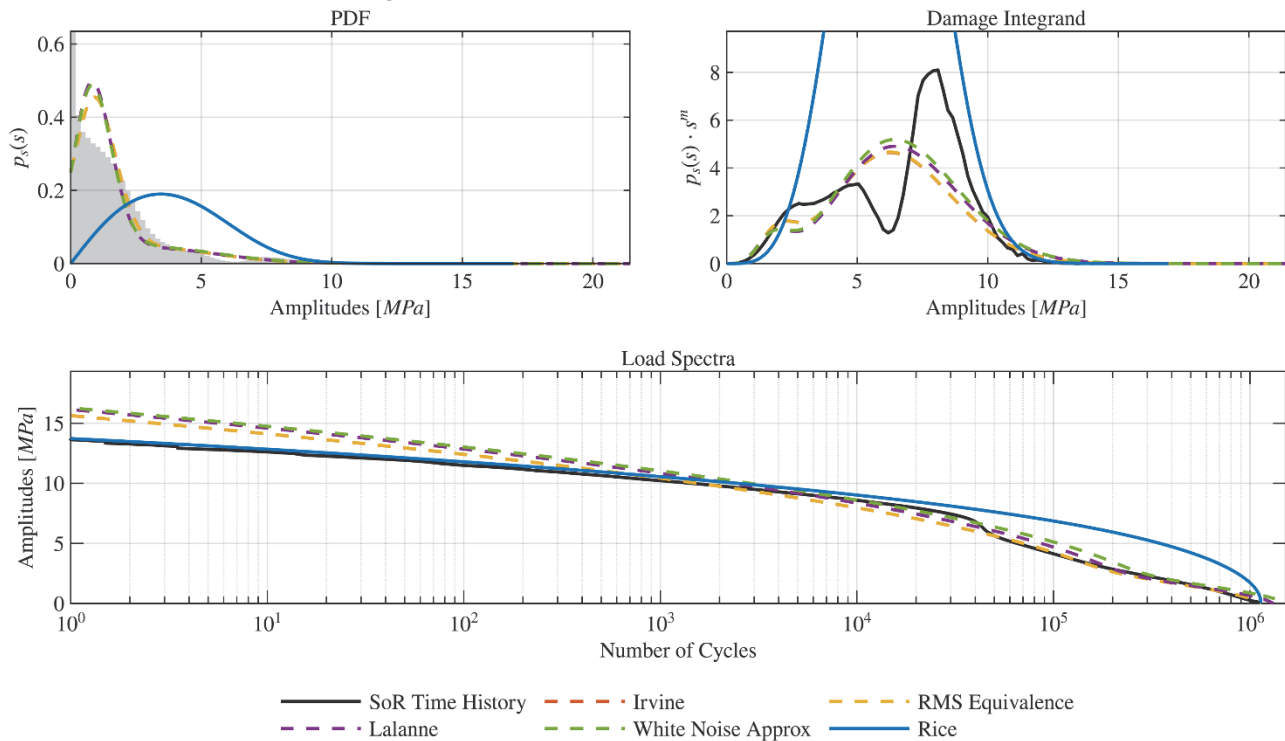


Figure 19: Statistical comparison of the predictive models under MIL-STD 810 SoR loading. The reference RFC probability density function is depicted as a shaded grey histogram in the first panel.

CONCLUSIONS AND PRACTICAL RECOMMENDATIONS

This study presented a comprehensive comparative analysis of spectral methods for estimating fatigue damage under SoR loading conditions, evaluating both the theoretical approach of Rice and the practical Equivalent PSD methods. The performances of these predictive models were systematically benchmarked against exact time-domain calculations, derived from the application of the RFC algorithm over 3600s synthetic time-histories. Assuming a relative



pseudo-damage index with typical fatigue parameters ($m=3$, $C=1$), the evaluation framework rigorously assessed both the percentage error on the expected damage and the statistical representativeness of the underlying cycle distributions, with a specific focus on the actual damage integrand. These numerical investigations, conducted across narrowband, wideband, and multi-sine MIL-STD-810 scenarios, highlighted the distinct predictive capabilities and inherent operational limits of each methodology.

What emerges from the presented review is that the selection of the most appropriate fatigue assessment method must be guided by the spectral characteristics of the loading environment. According to the considered loading environments and to the assumptions on Q and m the following conclusions can be drawn:

- Considering its theoretical characteristics and the results of the numerical simulations, Rice's method is regarded as the optimal and mathematically rigorous choice for in-band resonant excitations. Results demonstrate that the method is capable of great precision, but it is hardly adaptable to real loading environments, where multiple sine tones are simultaneously present, and the integration of Eqn. (21) tends to become computationally expensive.
- For broader industrial applications and standardized qualification profiles involving wideband noise, spectral separation and more than one sinusoid, Equivalent PSD methods seem to provide a highly effective tool for reliable damage evaluation. In the specific case of $m=3$ (and considering a value of $Q=10$ for each approach that requires SDOF dynamics), Irvine and RMS Equivalence methods demonstrate very similar behaviours and close estimates. This indicates that, potentially, these approaches can be selected regardless of the NB or WB nature of the random component, and for any number of sinusoids. Due to their accuracy and easy implementation, these methods are recommended mainly in early design and optimisation phases. Their computational efficiency and the apparent applicability to general loading environments allow for the rapid analysis of entire complex components, accurately identifying the most damaged critical areas.
- Lalanne's iterative method seems to yield better estimates when the random component is WB and the frequency ratio k is close to 1 (spectral coincidence). The low error observed in the MIL-STD scenario seems to suggest that this approach behaves in a better way whenever there are multiple sinusoids. The method may thus be applied to scenarios that include realistic standardized profiles. Since the Lalanne approach always appears to provide conservative damage index estimates, it may be employed whenever a safety-oriented design approach is required, to ensure that no load combination leads to failure, accepting a degree of component over-design.
- The White Noise Approximation appears to produce minor errors whenever the harmonic frequency coincides with the central random frequency, and the random noise band is narrow. In the MIL-STD scenario, instead, the error is the highest amongst the PSD equivalence methods, at least for the considered assumptions.

For the final certification of the component, the time-domain RFC method remains indispensable. However, its application can be restricted solely to the critical elements previously identified through spectral methods, thereby optimizing computational resources.

According to previous considerations, Tab. 1 summarizes and compares all the available SoR fatigue assessment approaches, serving as a practical guide for engineers facing SoR fatigue damage calculation.



	Rice's PDF	RMS Equivalence	Irvine's method	Lalanne's method	White Noise Approximation
Underlying assumptions	Random component is NB; Incommensurable frequencies; Harmonic frequencies are within the random band.	Random background and harmonic components are independent processes.	Two signals that cause the same damage to an SDOF system, for a given Q value, are also equally damaging towards every system with the same Q value.	Two signals with the same FDS, for a given Q value, are also equally damaging towards every system with the same Q value.	Same as Lalanne's; The local approximation of a PSD to white noise.
Computational Complexity	Low when the number of sinusoids equals 1. High otherwise.	Extremely low.	Low.	Low.	Extremely Low.
Required Input Parameters	Random PSD; Sinusoidal amplitudes and frequencies.	Random PSD; Sinusoidal amplitudes and frequencies; Sine to NB random conversion band Δf .	Random PSD; Sinusoidal amplitudes and frequencies; SN curve's slope m and intercept C ; SDOF system's quality factor Q ; Sine to NB conversion band Δf .	Random PSD; Sinusoidal amplitudes and frequencies; SN curve's slope m and intercept C ; SoR Fatigue Damage Spectrum.	Random PSD; Sinusoidal amplitudes and frequencies; SN curve's slope m and intercept C ; SoR FDS; Quality factor Q used for the FDS evaluation.
Main Advantages	Potentially high statistical fidelity; Provides a stress cycle pdf; High accuracy within underlying hypothesis.	Does not require system dynamics; Does not depend on m nor Q ; Fast implementation; Allows the direct use of spectral method.	Fast implementation; Allows the direct use of spectral method.	Fast implementation; Allows the direct use of spectral method.	Extremely fast implementation; Allows the direct use of spectral method.
Principal Limitations	Restricted domain of validity; Too conservative if outside of the assumptions; Long computational times for more than one sinusoid.	Depends on the choice of Δf ; Different Δf values give different equivalent PSDs.	Depends on m ; Depends on the choice of Δf ; Different Δf values give different equivalent PSDs. Requires the Q value of an SDOF system.	Depends on m ; Need to evaluate the SoR FDS. Requires the Q value of the set of SDOF systems used for the FDS evaluation.	Depends on m ; Need to evaluate the SoR FDS; Requires the Q value of the set of SDOF systems used for the FDS evaluation.
Recommended Applications	Every environment that falls under the underlying assumptions. Load profiles with a single harmonic.	Design and optimisation; Any random bandwidth; Any number of sinusoids.	Design and optimisation; Any random bandwidth; Any number of sinusoids.	Design for safety; WB random background; k values close to unity; Multiple sinusoids.	k values close to unity; Other methods are preferred for realistic MIL-SDT-like profiles.

Table 1: Summary table for direct comparison of the SoR fatigue assessment approaches (based of the selected values of Q and m).



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NOMENCLATURE

a	Sinusoidal Amplitude
a_s	Sinusoidal Stress Amplitude
a_{resp}	Amplitude of the Sinusoidal Response of an SDOF System according to Irvine's Method
C	Intercept Parameter of an SN Curve
df	Frequency Resolution of a PSD
D	Palmgren-Miner's Linear Cumulative Damage
D_{sine}	Damage Evaluated on the SDOF System Sinusoidal Response according to Irvine's Method
D_{start}	Damage Evaluated on the SDOF System Response to $G_{NB \text{ start}}$
$E[\]$	Expected Value Operator
f	Frequency
f_0	Natural Frequency
f_r	Random PSD Central Frequency
f_{rot}	Main Rotor Frequency
f_s	Sinusoidal Frequency
${}_1F_1$	Hypergeometric Function
FDS	Fatigue Damage Spectrum
FDS_0	FDS derived from G_0
FDS_{target}	Target Sine on Random FDS for Lalanne's Method
$\mathcal{FT}[\]$	Fourier Transform Function
G_0	First Guess PSD for Lalanne's Method
G_{NB}	Final Equivalent NB PSD Level according to Irvine's Method for Sine to Random conversion
$G_{NB \text{ start}}$	Initial Equivalent NB PSD Level according to Irvine's Method for Sine to Random conversion
G_r	Random Component PSD according to the RMS Equivalence Method
$G_{s\&R}$	Random Gaussian PSD Equivalent to a Sine on Random Signal
G_{xx}	Single-Sided Power Spectral Density
G_{xx}^-	Single-Sided PSD of a Stationary Gaussian Random Acceleration Signal
b	Convolution Integral Integration Variable
$H_{z/x}^-$	Frequency Response Function between Input Acceleration and Output Relative Displacement
I_0	Modified Bessel Function of Order Zero
J_0	Bessel Function of Order Zero
k	Frequency Ratio
K	Proportionality Factor Between Relative Response Displacements and Stresses
m	Slope Parameter of an SN Curve
m_n	Spectral Moment of n -th Order
n_b	Number of Blades
n_i	Number of Cycles at amplitude $Z_{p,i}$



N	Number of Fatigue Cycles
N_{bin}	Number of Amplitude Bins
N_h	Number of Harmonic Components in a SoR Signal
p_s	PDF of Rainflow Counted Stress Amplitudes
p_{s_p}	PDF of Stress Peak Values
p_x	PDF of Instantaneous Values of a Random Signal
p_{x_p}	PDF of Peak Values of a Random Signal
p_y	PDF of the Instantaneous Values of a Sinusoidal Signal
p_{y_p}	PDF of Peak Values of a Sinusoidal Signal
p_z	PDF of the Instantaneous Values of a Sine on Random Signal
p_{z_p}	PDF of the Peak Values of the Relative Displacement Response of an SDOF System
p_ζ	PDF of Normalized Maxima of a Random Signal
PD	Pseudo Damage
PD_{RFC}	Pseudo Damage evaluated from RFC Counted Cycles
Q	Quality Factor
R_{xx}	Autocorrelation Function
s	Stress Cycle Amplitude
s_{inst}	Stress Instantaneous Values
s_p	Stress Peak Values (Maxima)
S_{xx}	Double-Sided Power Spectral Density
t	Time
T	Duration of a Signal
x	Stationary Gaussian Random Signal
x_p	Peak Values (Maxima) of a Stationary Gaussian Random Signal
y	Sinusoidal Signal
y_p	Peak Values (Maxima) of a Sinusoidal Signal
z	Sine on Random Signal
Z	Relative Displacement Response of an SDOF System
Z_p	Peak Values (Maxima) of an SDOF System's Relative Displacement Response
$Z_{S,RMS}$	Root Mean Square Value of the Sinusoidal Relative Displacement Response
$\dot{Z}_{S,RMS}$	Root Mean Square Value of the Velocity of the Sinusoidal Relative Displacement Response
$\ddot{Z}_{S,RMS}$	Root Mean Square Value of the Acceleration of the Sinusoidal Relative Displacement Response
α_0	Ratio between Sinusoidal and Random Responses
γ	Irregularity Factor
$\Gamma()$	Gamma Function
$\delta()$	Dirac Delta Function
Δf	Frequency Band used for Sine to PSD Conversion
ϵ	Bandwidth Parameter
ζ	Normalized Maxima of a Random Signal
η	Transform Variable associated with the Characteristic Function
θ	Argument of a Sinusoidal Function
μ_x	Mean Value of a Stationary Gaussian Signal



ν_0^+	Number of Zero-Upcrossings per second
ν_p	Peak Rate
ν_{pSoR}	Peak Rate of a Sine on Random Signal
ξ	Damping Ratio
σ_x	Standard Deviation of a Zero Mean Stationary Gaussian Signal
$\sigma_{s,r}$	Root Mean Square Value of the Random Stress Component
σ_{SoR}	Root Mean Square Value of the SoR Signal
$\sigma_{Z,r}$	Root Mean Square Value of the Random Component of the Relative Displacement Response
σ_Z	Root Mean Square Value of an SDOF System's Relative Displacement Response
$\sigma_{\dot{Z}}$	Root Mean Square Value of the Velocity of an SDOF System's Relative Displacement Response
$\sigma_{\ddot{Z}}$	Root Mean Square Value of the Acceleration of an SDOF System's Relative Displacement Response
ϕ	Characteristic Function of a PDF
ϕ_x	Characteristic Function of the PDF of a Gaussian Random Signal
ϕ_y	Characteristic Function of the PDF of a Sinusoidal Signal
ϕ_x	Characteristic Function of the PDF of a Sine on Random Signal
ψ	Phase of a Sinusoidal Signal
ω	Angular Frequency
ω_0	Natural Angular Frequency
ω_s	Angular Frequency of a Sinusoidal Signal